ANL-Gen-IV-078

OSMOSE Program: Statistical Review of Oscillation Measurements in the MINERVE Reactor R1-UO2 Configuration

Topical Report

Nuclear Engineering Division

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OSMOSE Program: Statistical Review of Oscillation Measurements in the MINERVE Reactor R1-UO2 Configuration

Topical Report

by
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1. Introduction

The OSMOSE program is a collaboration on reactor physics experiments between the United States Department of Energy and the France Commissariat à l'Energie Atomique. At the working level, it is a collaborative effort between the Argonne National Laboratory and the CEA Cadarache Research Center.

The objective of this program is to measure very accurate integral reaction rates in representative spectra for the actinides important to future nuclear system designs, and to provide the experimental data for improving the basic nuclear data files. The main outcome of the OSMOSE measurement program will be an experimental database of reactivity-worth measurements in different neutron spectra for the heavy nuclides. This database can then be used as a benchmark to verify and validate reactor analysis codes. The OSMOSE program (Oscillation in Minerve of isotopes in Eupraxic Spectra) aims at improving neutronic predictions of advanced nuclear fuels through oscillation measurements in the MINERVE facility on samples containing the following separated actinides : ^{232}Th, ^{233}U, ^{234}U, ^{235}U, ^{236}U, ^{238}U, ^{237}Np, ^{238}Pu, ^{239}Pu, ^{240}Pu, ^{241}Pu, ^{242}Pu, ^{241}Am, ^{243}Am, ^{244}Cm, and ^{245}Cm **[1]**.

The first part of this report provides an overview of the experimental protocol and the typical processing of a series of experimental results which is currently performed at CEA-Cadarache.

In the second part of the report, improvements to this technique are presented, as well as the program that was created to process oscillation measurement results from the MINERVE facility in the future.

2. Experimental protocol and statistical review

2.1. The oscillation technique

2.1.1. Principle

This technique consists in oscillating samples that contain the studied actinide in the center of the experimental lattice in order to measure the associated reactivity variation. The uncertainty of this measurement, due to the reproducibility of the experiment, is proven to be lower than 1% **[3]**. Each sample is placed in an oscillation rod and moved periodically and vertically between two positions located in and out of the experimental zone as shown in Figure 1.

The studied sample is compared to a reference sample that differs only by the lack of actinide and that is placed in the bottom of the oscillation rod. Each sample is measured at least 4 times in order to significantly decrease systematic errors. A measurement corresponds to 10 oscillations of 120 seconds each.

The variation in flux induced by the oscillation is detected by a fission chamber placed in the driver zone, called the pilot chamber, which is servo-driven by a rotary automatic pilot rod. The pilot rod uses cadmium sectors, as shown in Figure 2, to compensate for the reactivity variations.

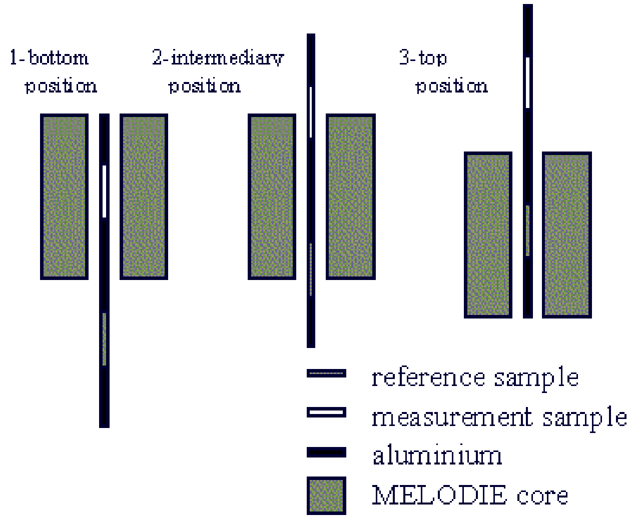


Figure 1: Movement of the oscillation sample in MINERVE

Figure 2: Rotor and stator of the automatic pilot rod

The pilot rod is calibrated using ^{235}U and ^{10}B samples, whose reactivity worth is known with uncertainties better than 1% through deterministic calculations.

Taking into account the uncertainties on the measurement (~1%), the samples (~2%), and the calibration of the pilot rod (~2%), the final experimental accuracy on the reactivity worth is about 3%.

2.1.2. The oscillation channel

The oscillation channel is a vertical electro-mechanical device (Figure 3), which is servo-driven by a position mechanism, whose characteristics are: square, pseudo square or sinusoidal movement; 900 mm stroke with selection of the mean position; sinusoidal period from 10 to 120 sec; square period from 20 to 120 s; square transit time of 1 sec; and sinusoidal transit time of 5 sec.

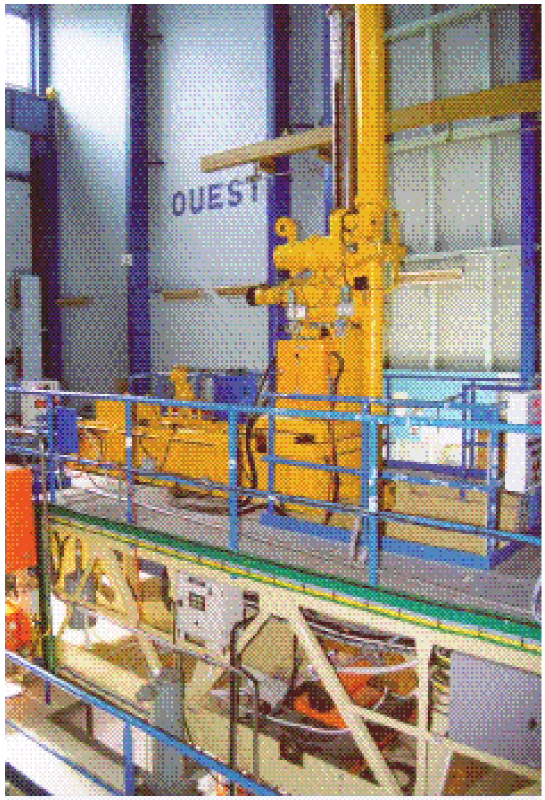


Figure 3: Oscillation channel of MINERVE

The oscillation is controlled by a clock, which provides synchronization signals and also sends them to the acquisition system. In the following study, the pseudo-square signals are preferred.

2.1.3. The automatic pilot rod

The pilot rod of MINERVE is a servo-driven system that rotates Cadmium sections in overlapping patterns (Figure 2) to cause a change in the neutron absorption of the pilot rod as a function of the angle of the rotor. The reactivity worth of the pilot rod is minimal when the sectors fully overlap, and maximal when they do not overlap (Figure 4).

The technique does not determine the absolute value of reactivity for a given rotor position, but instead is based on the relative reactivity effect, which is significantly more accurate for determining small changes in reactivity.

The automatic pilot rod is coupled with a Boron ionization chamber placed in the reflector through a measuring chain controlled by the neutron flux variations caused by the oscillations. A captor enables the recording of the rotation angle of the rotor (and so the superposition of the Cadmium sections) in the form of an analog voltage.

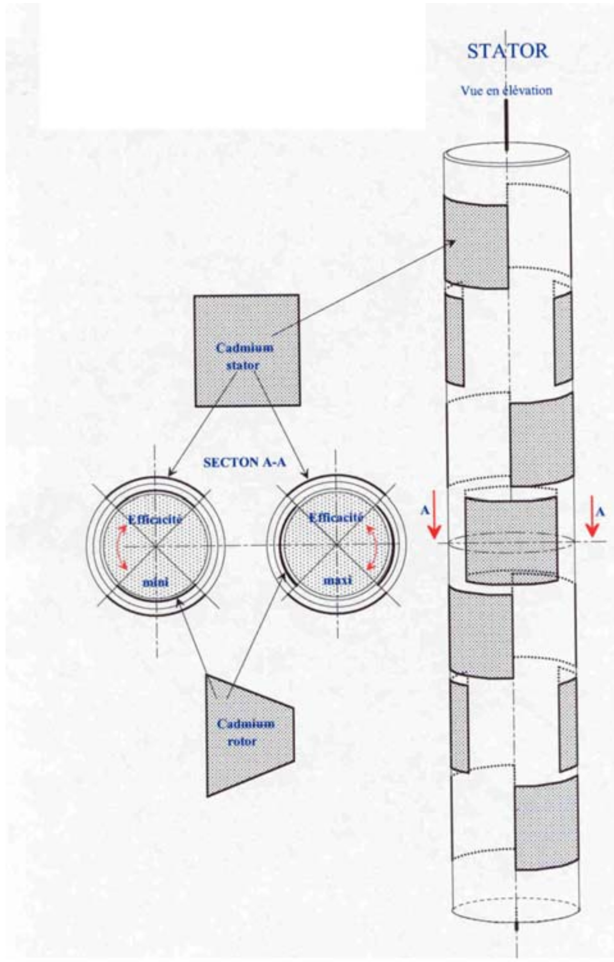


Figure 4: Superposition of Cadmium sheets of the automatic pilot rod

2.1.4. Calibration of the automatic pilot rod

Because of the overlapping Cadmium sections and the rotation of the Cadmium sections, the effect on reactivity is not proportional to the rotor position for all angles of rotation. The calibration of the pilot rod is necessary to determine the range of angles of rotation of the rotor that are proportional to reactivity (reactivity curve), and to accurately determine the differential change in reactivity (differential efficiency curve).

2.1.4.1. Reactivity curve of the pilot rod

To calibrate the pilot rod for oscillation measurements, the first stage dealt with verifying that the reactivity range of the pilot rod matches the range of the sample reactivity, i.e. ± 0.0001 (10 pcm). This was accomplished by positioning the pilot rod at different angles (i.e. different values of voltage on the rotor) and measuring the reactivity excess of the core. By doing this over the entire range of angles, a calibration curve of the pilot rod is created, as shown in Figure 5. This is a crude calibration that is adequate for initial positioning of the pilot rod but not sufficient for detailed measurements of small reactivity changes.

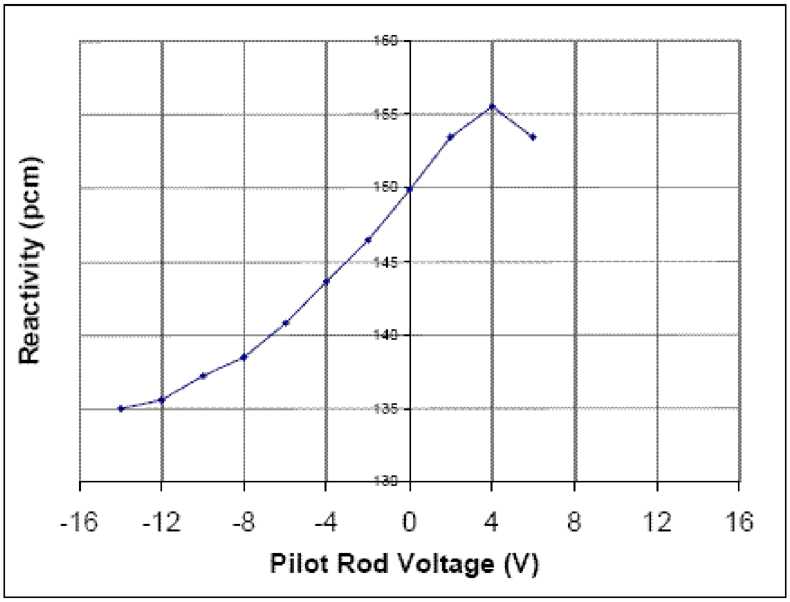


Figure 5: Reactivity curve of the pilot rod in the R1UO2 configuration

2.1.4.2. Differential efficiency curve of the pilot rod

The differential efficiency curve is the variation of the pilot rod angle (Δθ) induced by a fixed small variation of reactivity (Δρ) around the pilot rod angle θ. It is expressed by f(θ) = Δρ/Δθ, and processed as DEM1 for the angle θ_0 in MINERVE. The differential efficiency curve of the pilot rod is shown in Figure 6.

On an appropriate pilot rod angle, the differential efficiency curve is linear and can be written as:

$$f(\theta) = f(\theta_0) \times (1 + K \times (\theta - \theta_0))$$

where θ_0 is a reference mean angle (chosen in the middle of the linear part of the differential efficiency curve) and K depends on the slope of f(θ) and on θ_0 **[2] [4].** The angle of the pilot rod is measured in arbitrary pilot units, and processed as DEM2 in MINERVE.

Assuming a reference mean angle θ_0 = -700000 p.u., the constant K deduced from Figure 5 for the R1UO2 configuration is K= -8.939 x10^{-7}.

2.1.4.3. Theoretical approach of the correlation for the differential efficiency curve

The graph in Figure 6 shows the differential efficiency curve of the pilot rod obtained in MINERVE using an inverse linear fitting function for f(θ). The origin of the correlation is described to propose another way to deal with the signal from the pilot rod, based on the following parameters:

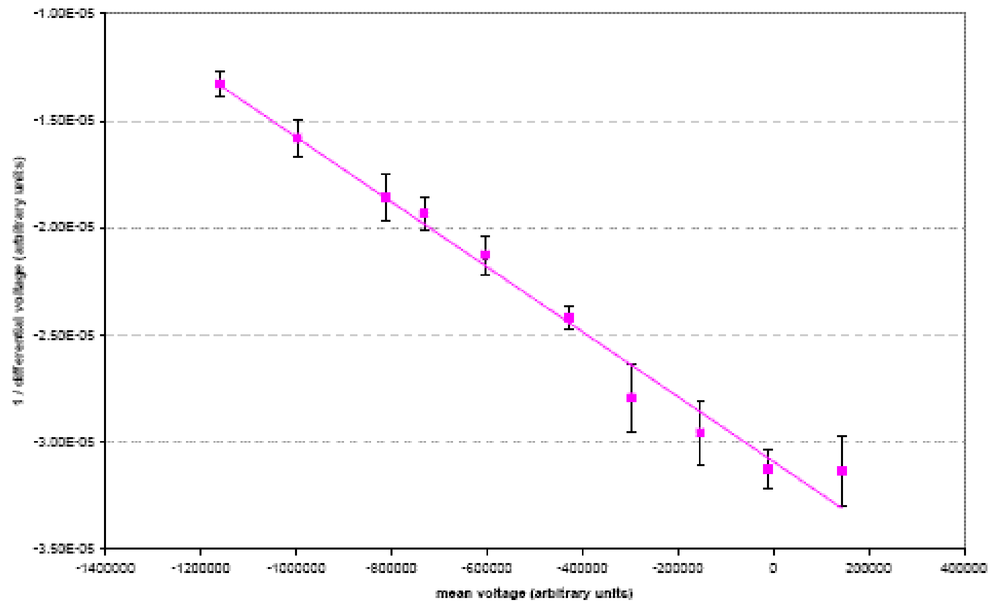


Figure 6: Differential efficiency curve of the pilot rod in the R1UO2 configuration using an inverse linear fitting function

θ_0 = reference angle to determine, which will then be chosen as the mean angle of the pilot rod for every oscillation measurement

θ = *DEM2* = mean angle set for the pilot rod at the beginning of the measurement of a given sample, such that $\theta \neq \theta_0$

$f(\theta)$ = *DEM1* = experimental amplitude obtained for a given sample

$f(\theta_0)$ = *DEM1** = amplitude which would have been obtained if the angle of the pilot rod had been experimentally set to $\theta = \theta_0$ (* stands for corrected)

A linear fit is performed in order to determine the value $f(\theta_0)$ based on $f(\theta)$. This can be written as follows:

$$f(\theta_0) = f(\theta) \times (1 - C_B \times (\theta - \theta_0)) \Leftrightarrow f(\theta) = \frac{f(\theta_0)}{(1 - C_B \times (\theta - \theta_0))}$$

where C_B is a constant to determine **[2]**.

Since $\theta \neq \theta_0$, a Taylor expansion to the first order gives:

$$f(\theta) \approx f(\theta_0) \times (1 + C_B \times (\theta - \theta_0))$$

For $\theta \neq \theta_0$, the Taylor series of the f function is also:

$$f(\theta) \approx f(\theta_0) + \left(\frac{\partial f}{\partial \theta}\right)_{\theta=\theta_0} \times (\theta - \theta_0)$$

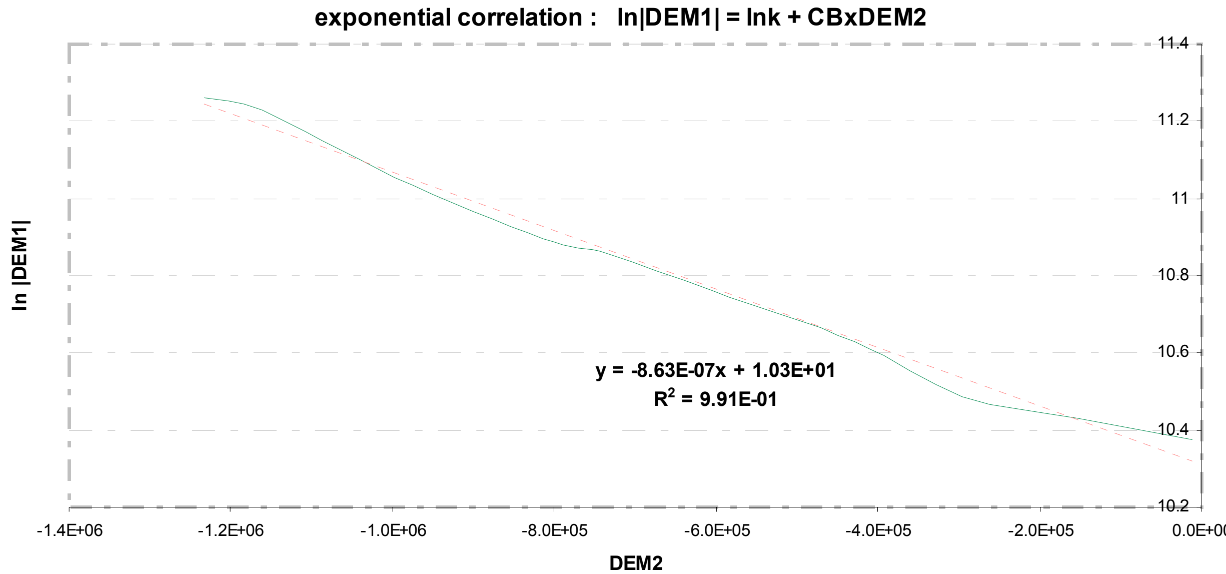


Figure 7: Differential efficiency curve of the pilot rod in the R1UO2 configuration using an exponential fitting function

As a result:

$$\left(\frac{\partial f}{\partial \theta}\right)_{\theta=\theta_0} = C_B \times f(\theta_0)$$

Although the previous equation is valid, it suggests an exponential fitting function such as:

$$f(\theta) = k \times \exp(C_B \times \theta)$$

Taking the positive value of f(θ) in the previous equation, leads to:

$$\ln(f(\theta)) = k' + C_B \times \theta \Leftrightarrow \ln|DEM1| = k' + C_B \times DEM2$$

This fitting function is plotted in Excel to perform a linear regression and determine the coefficient C_B, based on the calibration measurements provided by CEA (Figure 7).

The coefficient of linear regression provided by Excel, R^2, is still close to 1, although its value should be better in order to assess the hypothesis of an exponential fitting function. Notice that the value of the slope, $C_B = -8.63 \times 10^{-7}$, is satisfying compared to the previous one.

2.1.4.4. Conclusions about the correlation for the differential efficiency curve

The fact that the CEA performed a linear fitting from the experimental amplitude instead of an exponential fitting can be explained because only the first fitting function is implemented on the data processing system of the MINERVE facility. Besides, it has been argued that introducing more subtle functions would just increase the uncertainties on the correlation. This point cannot be neglected, since the regression coefficient obtained in the exponential case is not that good. Therefore, we will apply a linear correction with $C_B = -8.939 \times 10^{-7}$ to the signal provided by the automatic pilot rod during the measurements in the following study.

2.1.5. Collecting data from the experiments

The oscillation technique involves an analysis of periodic signals in the form of analog voltages, which represent the phenomena. The signals corresponding to the rotation angle of the pilot rod, the position of the oscillation rod, and the signal from the pilot chamber (respectively in red, yellow and blue on Figure 8) are synchronized by the control clock of the oscillation device, and processed in real time by the acquisition system, composed of a micro-processor and an acquisition card with analog-to-digital converters **[4]**.

Figure 9 shows an example of the signal of the pilot rod resulting from this processing. Notice the prompt jump and prompt drop phenomena, due to the oscillation mechanism: the sample suddenly disappears from the experimental zone, is transitorily replaced by the aluminum spacer and then by the other reference sample placed in the oscillation channel. Notice also that the stability mechanism of the control chain affects the signal in the return to the mean stage value, especially on the second upper stage.

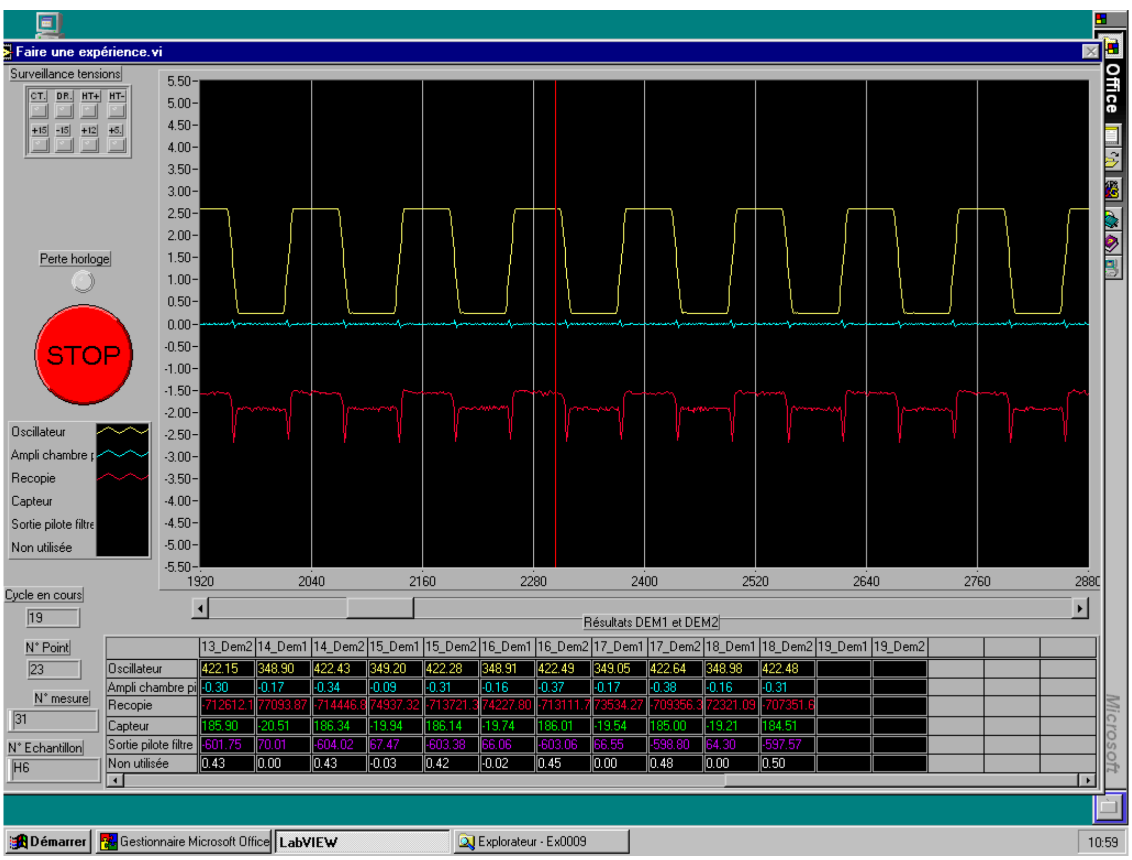


Figure 8: Example of an acquisition signal during an oscillation measurement

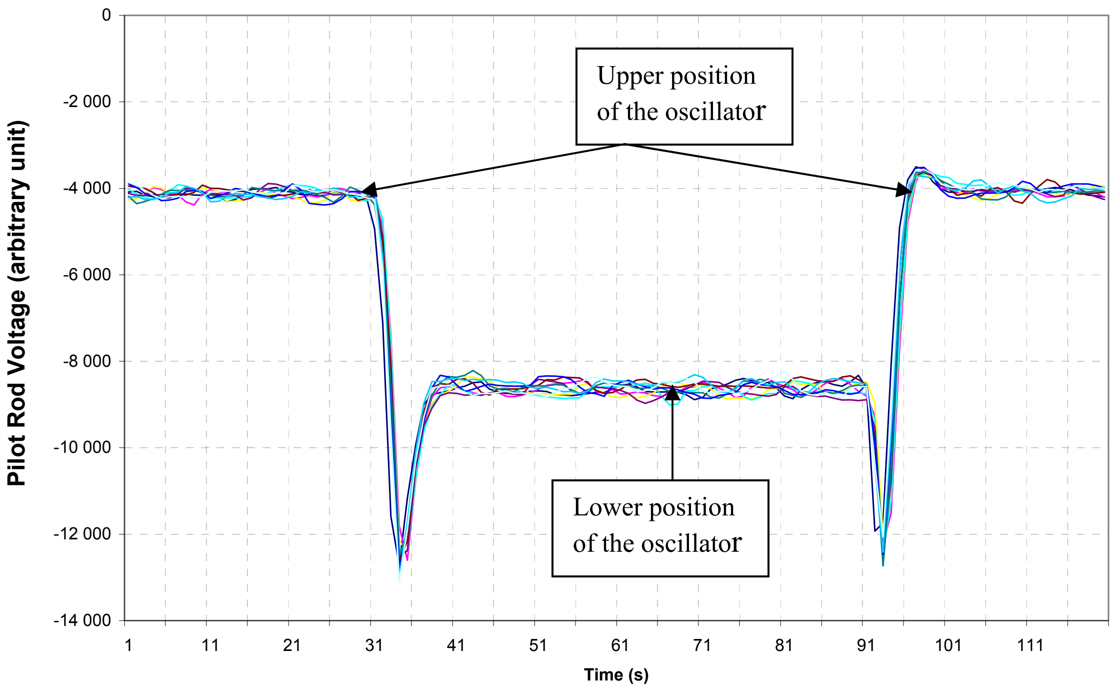


Figure 9: Example of a signal from the pilot rod for pseudo-square oscillations

2.2. Statistical review of the experimental data

2.2.1. Structure of the measurements

For each sample, at least five measurements are taken. Each measurement consists of a series of 10 or 20 cycles of oscillation of the sample. Let us consider that each measurement has 20 cycles.

The mean amplitude of the pilot rod is determined for each cycle of oscillation. This value is noted as:

$$A_{cij} \ , i \in \{1..n\}, j \in \{1..20\}$$

where i is the number of the measurement, out of n measurements and j is the number of the cycle.

The comparison of the amplitude for each cycle within one measurement provides information on the repeatability of the oscillations. This review will be performed in the following section.

The amplitudes of the 20 cycles are then averaged to obtain the mean amplitude of a given measurement, noted as A_{ci}. The comparison of the mean amplitude for all measurements of the same sample provides information on the reproducibility of the measurement.

The mean amplitudes are finally averaged over all of the measurements to determine the mean amplitude for a given sample, noted as A_c. The mean amplitude for a sample is then compared with the values from the calibration samples to determine the reactivity effect of the sample **[2]**.

2.2.2. Study of the stability in positioning of the oscillator on a given cycle

To assess the stability in positioning of the oscillator on a given cycle, j, of the measurement i, the following approach is followed.

2.2.2.1. Methodology

Each cycle is made of 120 measurement points. Because of the prompt drop and prompt jump phenomena, the stability stages are checked with points 1 to 30 and 111 to 120 for the upper stage (40 points) and with points 51 to 90 for the lower stage (also 40 points).

For each cycle, the mean amplitude is determined as:

$$A_{cij} = \sum_{k=1}^{40} (h_{kij} - b_{kij})$$

where A_{cij} is the amplitude of the cycle j of the measurement i; h_{kij} is the position signal of the oscillator for the point k of the upper stage of cycle j and measurement I; and b_{kij} is the position signal of the oscillator for the point k of the lower stage of cycle j and measurement i.

Considering that the points of upper and lower stages are uncorrelated, we can calculate the composed standard deviation of A_{cij} as follows:

$$\sigma(A_{cij}) = \sqrt{40} \times \sqrt{\sigma^2(h_{ij}) + \sigma^2(b_{ij})}$$

where $\sigma(h_{ij})$ and $\sigma(b_{ij})$ are the standard deviations of the points from the upper and the lower stage respectively.

2.2.2.2. Results

A previous study performed by CEA **[2]** on 12 measurements of 20 cycles has shown that this uncertainty is comprised between 0 and 111 (arbitrary unit) before the refurbishment of the oscillation device, and between 2 and 110 (a.u.) after refurbishment, whereas the mean amplitude was about 390000 a.u. This proves the excellent stability known at about 0.01% in both cases. Considering that this amplitude corresponds to a stroke of 700mm for the oscillation rod, we can conclude that the stability in positioning on a given cycle is better than 0.1mm.

2.2.3. Study of the repeatability in positioning of the oscillator on 20 cycles

After having checked the stability on a given cycle, the repeatability in positioning of the oscillator on all cycles (j from 1 to 20) of the measurement i is then assessed.

2.2.3.1. Methodology

A Chi-square test (χ^2-Test) is performed with a risk $\alpha = 5\%$ between the internal and the external standard deviations of the amplitude averaged on 20 cycles of a measurement, defined as follows:

$$\sigma_{\text{int}}^2(A_{ci}) = \frac{1}{\sum_{j=1}^{20} \frac{1}{\sigma^2(A_{cij})}}$$

and

$$\sigma_{ext}^2(A_{ci}) = \frac{\sum_{j=1}^{20} \frac{(A_{cij} - A_{ci})^2}{\sigma^2(A_{cij})}}{20 \times \sum_{j=1}^{20} \frac{1}{\sigma^2(A_{cij})}}$$

where A_{ci} is defined as:

$$A_{ci} = \frac{\sum_{j=1}^{20} \frac{A_{cij}}{\sigma^2(A_{cij})}}{\sum_{j=1}^{20} \frac{1}{\sigma^2(A_{cij})}}$$

The internal standard deviation corresponds to the quadratic reduction of the standard deviations on each individual measurement. The external standard deviation is relative to the amplitude of the measurement (i.e. averaged on all cycles), balanced by the statistical weight of the amplitude of each cycle. Therefore, $\sigma_{ext}(A_{ci})$ represents the uncertainty on the mean position of 20 cycles of a given measurement.

Thus, the uncertainty on repeatability associated with the mean position on an individual cycle is $\sqrt{20}$ x $\sigma_{ext}(A_{ci})$.

The χ^2-Test is defined with the following hypotheses:

H_0 : $\sigma_{ext}(A_{ci}) = \sigma_{int}(A_{ci})$
H_1 : $\sigma_{ext}(A_{ci}) > \sigma_{int}(A_{ci})$

The risk $\alpha = 5\%$ is the risk to conclude that $\sigma_{ext}(A_{ci})$ is superior to $\sigma_{int}(A_{ci})$, when H_0 is realized.

The following ratio is assessed:

$$\frac{\nu \times \sigma_{ext}^2(A_{ci})}{\sigma_{\text{int}}^2(A_{ci})}$$

where υ is the number of degree of freedom. It is equal to 19 here, since there are 20 cycles. When the previous ratio is superior to the threshold $\chi^2_{1-\alpha}$ given by a statistic table (Appendix 5), the test is significant. It is concluded that

$$\sigma_{ext}(A_{ci}) > \sigma_{int}(A_{ci})$$

when

$$\frac{\nu \times \sigma^2_{ext}(A_{ci})}{\sigma^2_{\text{int}}(A_{ci})} > \chi^2_{1-\alpha}$$

so that the repeatability in positioning of the oscillator on a given measurement cannot be fully explained by the stability on a cycle.

2.2.3.2 Results

The χ^2-Test was performed on 12 measurements of the VALMONT program at the CEA-Cadarache. Table 1 summarizes the results after the refurbishment of the oscillator.

Notice that the hypothesis H_0 cannot be accepted (except for measurement #9). Therefore, the uncertainty associated with the repeatability in positioning of the oscillator on the 20 cycles of a given measurement cannot be explained by the stability on a cycle.

The external standard deviation has to be retained to assess the repeatability in positioning. It is comprised between 7 and 37 au, corresponding to the range 0.01-0.07 mm in terms of absolute position. Multiplying this range by $\sqrt{20}$ gives the uncertainty on repeatability associated with the mean position on a given cycle, which is comprised between 0.04 mm and 0.31 mm.

Table 1
Results concerning the repeatability in positioning of the oscillator on 12 measurements of the VALMONT program after refurbishment

Measurement	σ_{ext}	σ_{int}	$\frac{\nu \times \sigma^2_{ext}(A_{ci})}{\sigma^2_{\text{int}}(A_{ci})}$	$\chi^2_{1-\alpha}$
1	24.5	4.9	469	30.1
2	26.6	5.1	512	30.1
3	21.6	5.5	289	30.1
4	18.2	5.6	202	30.1
5	37.4	4.8	1138	30.1
6	25.1	5.1	459	30.1
7	23.9	5.4	373	30.1
8	29.4	5.6	519	30.1
9	6.9	5.7	27	30.1
10	28.5	5.1	590	30.1
11	29.0	4.4	842	30.1
12	21.7	5.3	313	30.1

2.2.4 Study of the reproducibility in positioning of the oscillator on all measurements

After having checked the stability on a given cycle and its repeatability, the reproducibility in positioning of the oscillator on all measurements is then assessed.

2.2.4.1. Methodology

A similar one-tailed Chi-square test (χ^2-Test) with a risk of α = 5% is performed between the internal and the external standard deviations of the amplitude averaged on the 12 measurements of the experiment, defined as follows:

$$\sigma^2_{\text{int}}(A_c) = \frac{1}{\sum_{i=1}^{12} \frac{1}{\sigma^2(A_{ci})}}$$

and

$$\sigma^2_{ext}(A_c) = \frac{\sum_{i=1}^{12} \frac{(A_{ci} - A_c)^2}{\sigma^2(A_{ci})}}{20 \times \sum_{i=1}^{12} \frac{1}{\sigma^2(A_{ci})}}$$

where A_c is defined as:

$$A_c = \frac{\sum_{i=1}^{12} \frac{A_{ci}}{\sigma^2(A_{ci})}}{\sum_{i=1}^{12} \frac{1}{\sigma^2(A_{ci})}}$$

The internal standard deviation corresponds to the quadratic reduction of the standard deviations on each measurement. $\sigma_{ext}(A_c)$ represents the uncertainty on the mean position of a series of 12 measurements of 20 cycles. Thus the uncertainty on repeatability associated with the mean position of a given measurement is $\sqrt{12}$ x $\sigma_{ext}(A_c)$ and the uncertainty on repeatability associated with the mean position on a cycle of a given measurement is $\sqrt{12 \times 20}$ x $\sigma_{ext}(A_c)$.

The one-tailed χ^2-Test is defined with the following hypotheses: for H_0, $\sigma_{ext}(A_c) = \sigma_{int}(A_c)$; and for H_1, $\sigma_{ext}(A_c) > \sigma_{int}(A_c)$. The risk α = 5% is the risk to conclude that $\sigma_{ext}(A_c)$ is superior to $\sigma_{int}(A_c)$, when H_0 is accepted.

The following ratio is assessed:

$$\frac{\nu \times \sigma^2_{ext}(A_{ci})}{\sigma^2_{\text{int}}(A_{ci})}$$

where υ is the number of degree of freedom. It is equal to 11 here, since there are 12 measurements. When the previous ratio is superior to the threshold $\chi^2_{1-\alpha}$ given by a statistics table (Appendix 3), the test is significant. It is concluded that

$$\sigma_{ext}(A_{ci}) > \sigma_{int}(A_{ci})$$

when

$$\frac{\nu \times \sigma^2_{ext}(A_{ci})}{\sigma^2_{\text{int}}(A_{ci})} > \chi^2_{1-\alpha}$$

so that the reproducibility in positioning of the oscillator from one measurement to another cannot be fully explained by the repeatability in the signal on a given measurement.

2.2.4.2. Results

The χ^2-Test was performed on the same 12 measurements of the VALMONT program at the CEA-Cadarache. Table 2 summarizes the results after the refurbishment of the oscillator.

Notice that the hypothesis H_0 cannot be accepted either. Therefore, the external standard deviation has to be retained to assess the reproducibility in positioning. It is equal to 19 au, corresponding to 0.03 mm in terms of absolute position. Multiplying this value by $\sqrt{12}$ gives the uncertainty on reproducibility associated with the mean position on a given measurement, which is equal to 0.1 mm. Multiplying that value by $\sqrt{20}$ gives the uncertainty on reproducibility associated with the mean position on a given cycle, which is equal to 0.5 mm. This is the uncertainty associated to the stroke of the oscillation rod during any cycle. Notice that it is low (0.5%) compared to the length of the oscillated samples (~100 mm).

Table 2
Results concerning the reproducibility in positioning of the oscillator on 12 measurements of the VALMONT program after refurbishment

σ_{ext}	σ_{int}	$\frac{\nu \times \sigma^2_{ext}(A_{ci})}{\sigma^2_{\text{int}}(A_{ci})}$	$\chi^2_{1-\alpha}$
19.3	4.9	154	19.7

2.2.5. Analysis of the oscillation results

2.2.5.1. Statistical processing of the signal – Formalizing the standard deviation of a measurement

The first method of interpreting the experimental results is described in this section. The signal to process is the voltage from the automatic pilot rod, which is directly proportional to its rotation angle.

The causes of uncertainty on the signal are numerous **[6]**. Only the random part of the uncertainty will be considered. It can be separated into two independent terms, which are physically consistent - the statistical fluctuation in a measurement for a given loading of the oscillation rod and the fluctuation associated with the loading of the oscillation rod. The second term is different from one measurement to another.

Therefore, for a measurement campaign:

$$S_{ij} = S_0 + \delta_{cS_i} + \delta_{eS_{ij}}$$

where

$i \in \{1..n\}$ is the number of measurements of a given sample;
$j \in \{1..10\}$ is the number of the cycle in a given measurement;
S_{ij} is the signal of the sample in cycle j of the measurement j = amplitude of the signal from the pilot between the measured sample and the reference sample;
$\delta_{eS_{ij}}$ is the term representing the statistical fluctuation in a measurement, for a given loading (with standard deviation, σ_e); and
δ_{cS_i} is the term representing the fluctuation due to the loading (with a mean of zero and standard deviation, σ_l).

The following can also be defined:
$\overline{S_i} = \frac{1}{10} \times \sum_{j=1}^{10} S_{ij}$ the mean value of the signal on all cycles of the measurement i ;
$\overline{S} = \frac{1}{10 \times n} \times \sum_{ij} S_{ij} = \frac{1}{n} \times \sum_{i=1}^{n} \overline{S_i}$ the mean value of the signal on the n measurements.

Consequently, the following statistics will be used **[3] [5]**:
$\hat{\sigma}_l^2 = \frac{1}{n} \times \sum_{i=1}^{n} (\overline{S_i} - \overline{S})^2$ the standard deviation of the fluctuation term due to the loading ;
$\hat{\sigma}_e^2 = \frac{1}{10 \times n} \times \sum_{ij} (S_{ij} - \overline{S_i})^2 = \frac{1}{n} \times \sum_{i=1}^{n} \left[\frac{1}{10} \times \sum_{j=1}^{10} (S_{ij} - \overline{S_i})^2 \right]$ the standard deviation of the statistical fluctuation term in a measurement.
The total estimated standard deviation on the signal of a sample, $\hat{\sigma}_s$, is then given by:

$$\hat{\sigma}_s = \sqrt{\frac{\hat{\sigma}_e^2 + \hat{\sigma}_l^2}{n}}$$

This standard deviation is estimated based on the hypothesis that only the loading of the sample and the statistical fluctuation of the signal affect the uncertainty on the measurements. In order to qualify this generic standard deviation, we must check for each sample that $\hat{\sigma}_s$ accounts for all measurement uncertainties. A comparison with the common estimate standard deviation, s, will be performed through a χ^2-Test and is defined as follows:

$$s = \sqrt{\frac{1}{n-1} \times \sum_{i=1}^{n} (\overline{S_i} - \overline{S})^2}$$

The hypotheses of this test are:

H_0: s = $\hat{\sigma}_s$ $\hat{\sigma}_s$ is retained as the standard deviation on the measurements of a sample;
H_1: s > $\hat{\sigma}_s$ s is retained as the standard deviation on the measurements of a sample.

H_0 is accepted if $\frac{\nu \times s^2}{\hat{\sigma}_s^2}$ follows the $\chi^2_{1-\alpha}$ law with υ = n-1 degrees of freedom. We consider the risk α=5% that s > $\hat{\sigma}_s$ when H_0 is met. The threshold $\chi^2_{1-\alpha}$ is given by a statistical table. When $\frac{\nu \times s^2}{\hat{\sigma}_s^2}$ is superior to this threshold, the test is significant and the conclusion is s > $\hat{\sigma}_s$ **[3]**.

2.2.5.2. Methodology for processing a 10 cycle measurement

The previous method determines the standard deviation which accounts for all uncertainties of a given measurement. In the OSMOSE program, the studied samples were oscillated in ten cycle measurements. For each measurement, the following process is used for processing the data:

For importing and scrutinizing the data – (1) the signal values are copied from the raw data files and broken up into 10 columns representing the 10 cycles; (2) each value is multiplied by 4093 **[7]** in order to harmonize the ranges with the pilot unit specific to MINERVE ; (3) a simple algorithm sorts the points and rejects the irrelevant values due to the prompt drop and prompt jump phenomena ; and (4) the DEM1 and DEM2 values are also copied, corresponding to the starting angle of the pilot rod, and the mean reactivity worth of the sample compensated by the pilot rod.

For the calculations - the original DEM1 value is corrected according to the correlation in Section 1.1.4.3, the mean values of the amplitude on each cycle (A_{cij}) and on each measurement (A_{ci}) and finally for all measurements of a given sample (A_c) are determined. The estimators $\hat{\sigma}_e$, $\hat{\sigma}_l$, $\hat{\sigma}_s$ and s are also computed. Finally, the χ^2-Test estimator $\frac{\nu \times s^2}{\hat{\sigma}_s^2}$, and the threshold $\chi^2_{1-\alpha}$ are calculated which lead to retaining a generic standard deviation for each sample.

The results are presented in the next section and it appears that outliers can also be dealt with based on the proposed χ^2-Test. Consequently, some values are able to be rejected from a series of measurements of a given sample. This will be developed further and other statistical tests used to detect outliers out of experimental results will also be introduced.

3. Processing the data from OSMOSE measurements

An automatic data processing macro was developed in Excel to make a cross-comparison with the results from CEA concerning the first 50 measurements of the OSMOSE program. The macro proceeds in the same way as described in Section 2.2.5.2 and then additional tests are performed to detect outlying results.

3.1. Implementing the current method

3.1.1. Structure of the raw data

The output files from the acquisition system of MINERVE described in Section 2.1.5 are available. The experiment on the first series of OSMOSE samples is identified at CEA by the code FEX 47. All files therefore have a name like: 00470 + [number of measurement] + [extension]. They are stored in a folder structure described in Figure 10. The first file, which has a common Outlook extension, contains the parameters of the measurement. It is used to identify the name of the sample and the date of the measurement.

The operators of MINERVE use channel 2 of the acquisition system to record the voltage signals from the automatic pilot rod. First, they set the angle of the pilot rod at the beginning of each cycle of a measurement, which should be about -700000 au, as explained in Section 2.1.4. This value is stored as DEM2, and is found in the file with the ".v02" extension. The pilot chamber also returns a voltage representing the mean value of the reactivity due to the sample which is stored in the file with the ".v02" extension as well and is treated as DEM1 **[7]**.

Each sample of the first series has been oscillated at least 4 times in MINERVE. Each oscillation measurement counts 10 cycles of 120 seconds. The automatic pilot rod thereby returns 1200 values corresponding to the differential reactivity worth of the sample, which are stored in the ".Da2" file.

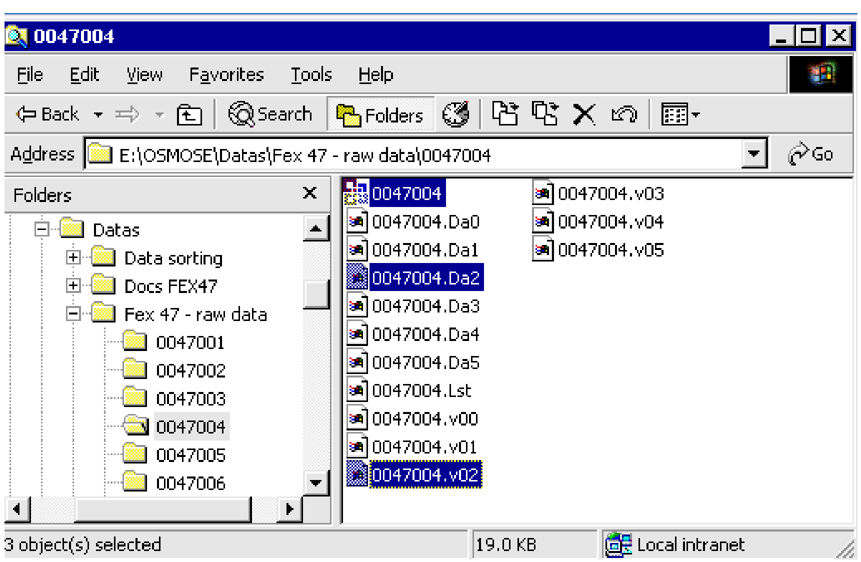


Figure 10: The structure of output files from MINERVE and the interesting files (highlighted)

3.1.2. Importing, sorting and scrutinizing the data

As described before (in 2.2.5.2), the data is imported into Excel and separated into 10 columns. In each column, the 120 values of the differential reactivity worth for each cycle are reported. DEM1 and DEM2 are also reported. Notice that the 120 values are multiplied by 4093 in order to standardize them with the pilot unit. The mean cycle is then calculated in an eleventh column.

In a second step, the data are scrutinized in order to reject the points that are not valuable for further processing. With a simple test based on the regularity of a mathematical continuous function, the points corresponding to the prompt jump and prompt drop phenomena and to the time delay of the controlling chain of MINERVE are removed from the data array. In fact, the original column is copied into a new one, where only the values corresponding to the flattest lower and upper stages as possible are retained.

An option allows the mean cycle of the considered measurement to be plotted in a separate worksheet. This is especially useful to see the effect of the data processing.

3.1.3. Calculating the output values and the statistical estimators of a measurement

From then on, the mean value of the signal, S_{cij}, is calculated for each cycle, as well as the amplitude between the upper and the lower stage, A_{cij}. As seen before, A_{cij} corresponds to the reactivity worth of the OSMOSE sample compared to the calibration sample placed in the bottom of the oscillation rod (Boron sample at 60 ppm, known as nb 8).

The DEM1* value is also calculated. This is the corrected value of DEM1, based on the correlation described in Section 2.1.4.3.

The macro then calculates the average values of the signal (S_{ci}), the amplitude (A_{ci}), and DEM1* (D_{ci}) for the 10 cycles for the considered measurement. The next step is to calculate the statistics for each measurement. These are the standard deviation for the 10 DEM1* values and the first sum of $\hat{\sigma}^2_{e_i}$.

3.1.4. Statistical review of all measurements of each sample

All measurements on MINERVE were performed in a cyclic way. Table 3 shows the sequence of all 50 measurements with associated dates and sample names. Notice that calibration samples are regularly oscillated in MINERVE to check the consistency of the results.

The first step of the review is to re-order these measurements sample by sample. This is performed in separate Excel worksheets for the values of A_{ci} (amplitude), D_{ci} (DEM1*) and S_{ci} (mean value of the signal). Since the following statistical analysis is the same for each output parameter, the focus is on the amplitude of a given sample.

First, the mean value of all measurements of the sample, A_c, is calculated. Based on this value, the macro calculates the estimators $\hat{\sigma}^2_{l_i}$ and $\hat{\sigma}^2_{e_i}$, the fluctuation term due to the loading and the statistical fluctuation term, respectively. The total estimated standard deviation term, σ_s^2, is then

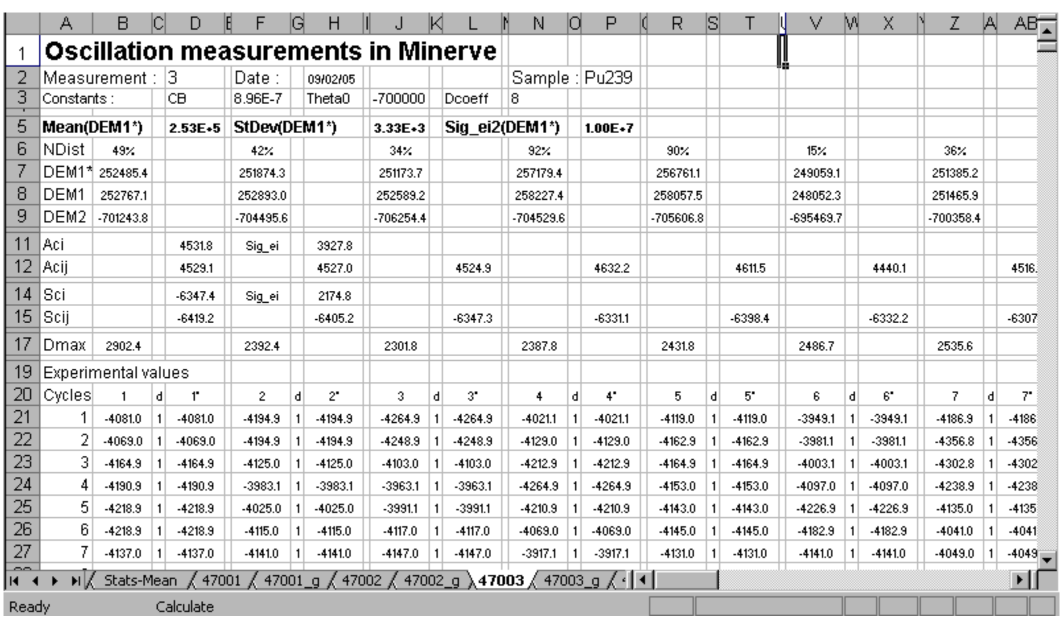
Table 3 The List of 50 Measurements – the first series of OSMOSE samples (FEX 47)											
#	Date	Sample	#	Date	Sample	#	Date	Sample	#	Date	Sample
1	9/1/05	Unat	14	9/13/05	Unat	27	9/29/05	U234	40	10/6/05	Unat
2	9/2/05	Th232	15	9/13/05	Th232	28	9/29/05	Ure	41	11/22/05	Np0.6
3	9/2/05	Pu239	16	9/19/05	Ure	29	9/29/05	H1	42	11/22/05	Np0.1
4	9/6/05	Pu242	17	9/20/05	Pu239	30	9/29/05	H3	43	11/22/05	No9
5	9/6/05	U234	18	9/20/05	Pu242	31	9/30/05	Unat	44	11/23/05	Np0.6
6	9/7/05	Ure	19	9/21/05	U234	32	9/30/05	Th232	45	11/23/05	Np0.1
7	9/8/05	Unat	20	9/21/05	Ure	33	9/30/05	Pu239	46	11/23/05	Np0.6
8	9/8/05	Th232	21	9/21/05	H1	34	9/30/05	Pu242	47	11/23/05	Np0.1
9	9/8/05	Pu239	22	9/27/05	H3	35	10/4/05	U234	48	11/23/05	No9
10	9/8/05	Pu242	23	9/27/05	Unat	36	10/4/05	Ure	49	11/24/05	Np0.6
11	9/12/05	U234	24	9/27/05	Th232	37	10/4/05	Unat	50	11/24/05	Np0.1
12	9/12/05	H1	25	9/28/05	Pu239	38	10/5/05	Ure			
13	9/13/05	H3	26	9/28/05	Pu242	39	10/5/05	Unat			

calculated, as well as the common estimate standard deviation term, s_i^2. Eventually, a χ^2-Test is performed according to Section 2.2.5.1 and the result is given in the last column.

3.1.5. Results and comments

3.1.5.1. Output files

The automatic data processing returns two output files: (1) the import file, where all the raw data is sorted and scrutinized, and (2) the statistical file, where the data are arranged sample by sample and tests are performed. Figure 11 is an excerpt of the import file, especially the headlines. Figure 12 shows the plot of a mean cycle, whereas Table 4 shows the statistical results for the natural U sample. The negative χ^2-Test result is explained in the next. section.



	A	B	C	D	F	G	H	J	K	L	N	O	P	R	S	T	V	W	X	Z	AA	AB
1	Oscillation measurements in Minerve																					
2	Measurement :			3	Date :		09/02/05				Sample :		Pu239									
3	Constants :			CB	8.96E-7		Theta0	-700000		Dcoeff	8											
5	Mean(DEM1*)			2.53E+5	StDev(DEM1*)			3.33E+3		Sig_ei2(DEM1*)			1.00E+7									
6	NDist	49%			42%			34%			92%			90%			15%			36%		
7	DEM1*	252485.4			251874.3			251173.7			257179.4			256761.1			249059.1			251385.2		
8	DEM1	252767.1			252893.0			252589.2			258227.4			258057.5			248052.3			251465.9		
9	DEM2	-701243.8			-704495.6			-706254.4			-704529.6			-705606.8			-695469.7			-700358.4		
11	Aci			4531.8	Sig_ei		3927.8															
12	Acij			4529.1			4527.0			4524.9			4632.2			4611.5			4440.1			4516.
14	Sci			-6347.4	Sig_ei		2174.8															
15	Scij			-6419.2			-6405.2			-6347.3			-6331.1			-6398.4			-6332.2			-6307
17	Dmax	2902.4			2392.4			2301.8			2387.8			2431.8			2486.7			2535.6		
19	Experimental values																					
20	Cycles	1	d	1*	2	d	2*	3	d	3*	4	d	4*	5	d	5*	6	d	6*	7	d	7*
21	1	-4081.0	1	-4081.0	-4194.9	1	-4194.9	-4264.9	1	-4264.9	-4021.1	1	-4021.1	-4119.0	1	-4119.0	-3949.1	1	-3949.1	-4186.9	1	-4186
22	2	-4069.0	1	-4069.0	-4194.9	1	-4194.9	-4248.9	1	-4248.9	-4129.0	1	-4129.0	-4162.9	1	-4162.9	-3981.1	1	-3981.1	-4356.8	1	-4356
23	3	-4164.9	1	-4164.9	-4125.0	1	-4125.0	-4103.0	1	-4103.0	-4212.9	1	-4212.9	-4164.9	1	-4164.9	-4003.1	1	-4003.1	-4302.8	1	-4302
24	4	-4190.9	1	-4190.9	-3983.1	1	-3983.1	-3963.1	1	-3963.1	-4264.9	1	-4264.9	-4153.0	1	-4153.0	-4097.0	1	-4097.0	-4238.9	1	-4238
25	5	-4218.9	1	-4218.9	-4025.0	1	-4025.0	-3991.1	1	-3991.1	-4210.9	1	-4210.9	-4143.0	1	-4143.0	-4226.9	1	-4226.9	-4135.0	1	-4135
26	6	-4218.9	1	-4218.9	-4115.0	1	-4115.0	-4117.0	1	-4117.0	-4069.0	1	-4069.0	-4145.0	1	-4145.0	-4182.9	1	-4182.9	-4041.0	1	-4041
27	7	-4137.0	1	-4137.0	-4141.0	1	-4141.0	-4147.0	1	-4147.0	-3917.1	1	-3917.1	-4131.0	1	-4131.0	-4141.0	1	-4141.0	-4049.0	1	-4049

Figure 11: Partial view of the output file after importing the raw data for a single measurement

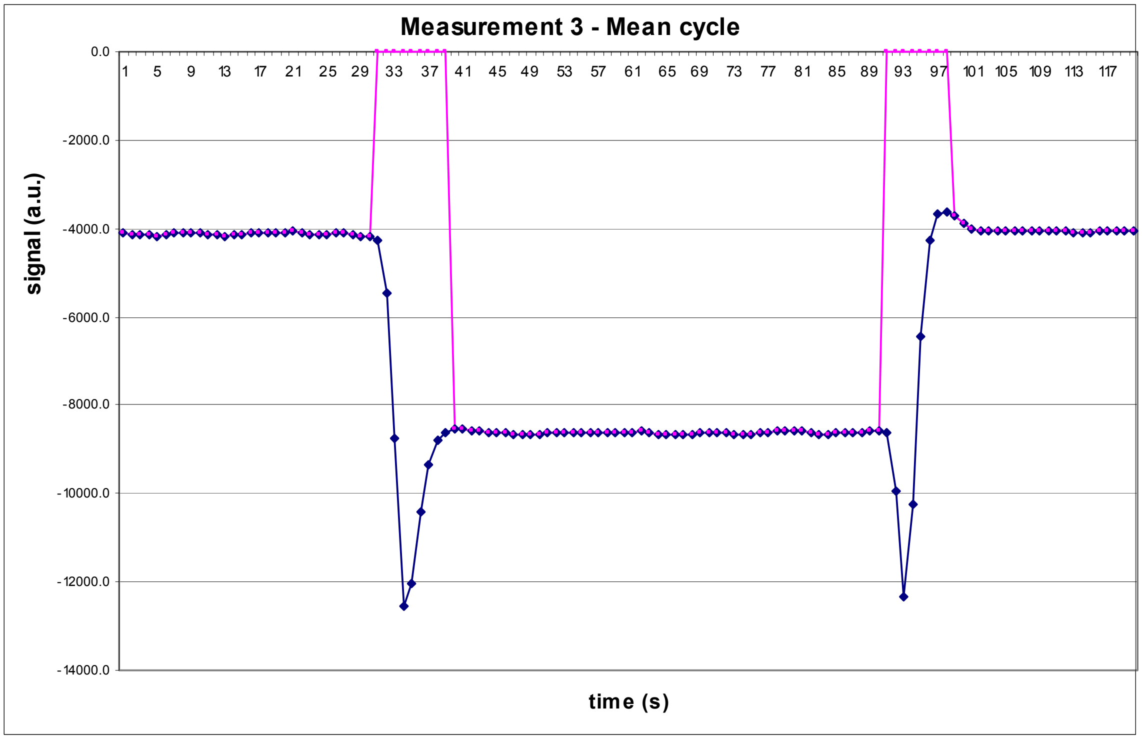


Figure 12: Mean cycle for Pu239 sample – data points before and after scrutinization

Table 4
Statistical review for the Unat sample : negative χ^2-Test results

Nb	in	Sig_ci2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi-inv	Test
1	1	7054	3838.9	1238.1	Unat	9/1/05	2752	545	35.33	14.07	Refused
7	1	10447	1317.2	1051.9	Unat	9/8/05					
14	1	257	1800.4	1138.1	Unat	9/13/05					
23	1	1330	1580.9	1190.6	Unat	9/27/05					
31	1	5	3730.2	1151.8	Unat	9/30/05					
37	1	113	1505.0	1143.4	Unat	10/4/05					
39	1	17	1249.8	1158.2	Unat	10/5/05					
40	1	44	610.7	1160.7	Unat	10/6/05					
	8	2408	1954	1154	Mean						

3.1.5.2. Cross-comparison with CEA results

CEA-Cadarache provided Excel files resulting from its own analysis of the oscillation measurements with the raw data files from MINERVE. The study that has been described in this report aims at making a cross-comparison with these results.

The review of all OSMOSE samples led to an outcome which was very similar to the conclusions of CEA-Cadarache. Appendix 1 summarizes the results obtained at ANL. Some measurements at the beginning of the experiment were considered as erroneous or at least inaccurate. Especially in the case of Unat, the first two measurements turn out to be outliers when considering the whole sample of 8 measurements. The results are similar for other samples whose χ^2-Test result was negative, such as Ure (Uranium from reprocessing). A closer look at the measurements led to the conclusion that the automatic pilot rod returned signals that were more regular after September 8, 2005 than before. It is assumed that the operators progressively got better at the loading of the oscillation rod. At least their technique became more consistent after September 8, 2005. This appears clearly in the suspected terms σ_l^2. Therefore, all results obtained prior to this date are questionable. CEA-Cadarache came to the same conclusion.

Nevertheless, although the values corresponding to the reactivity of each sample are consistent, there is a slight systematical difference from CEA results. We checked the reason for that and pointed out that the input values from the automatic pilot rod are different from the CEA values, for all measurements. There must be a filter in the acquisition system of MINERVE **[7]**. Other acquisition channels besides channel 2 report values with similar trends, which are related to the intermediary systems of the measuring control chain of the pilot rod (among others the signal from the Boron chamber dedicated to the pilot rod). This issue remains to be resolved until a proper investigation and understanding of the acquisition system of MINERVE or the acquisition protocol of its operators can be completed.

3.1.5.3. Conclusions

The main conclusion from this review of the oscillation results is that there is consistency between the ANL and CEA results. The method was successfully implemented but the input data should be checked in order to come to a point of resolution as to which values to use for DEM1 and DEM2.

It is also interesting to detect outlying values from a repetitive experiment such as the oscillation measurements **[7]**. The question is addressed in the next section.

3.2. Dealing with inconsistent data points

3.2.1. Common tests and main problems

Outliers in survey data are generally considered to be observations which are a long way from, or inconsistent with, the remainder of the data **[8]**. They are often the result of response or capture errors during collection. Outlier detection in surveys is commonly used to macro edit respondent data. This relieves the burden of excessive micro editing by detecting errors in data through the analysis of aggregate data **[9]**.

Most outlier detection methods use some measure of distance to evaluate how far away an observation is from the center of the data. To measure this distance, the sample mean and variance may be used but since they are not robust to outliers, they can mask the very observations we seek to detect. This is particularly true when dealing with small samples (in terms of number of measurements), which is the case for the OSMOSE measurements. To avoid this masking effect, robust scale and location estimators, which are inherently resistant to outliers, may be used. This is why many outlier detection methods use order statistics, such as the median or quartile.

Perhaps the most popular univariate outlier detection technique for survey data is the quartile method. This method creates an allowable range for the data using lower and upper quartiles: data falling outside of the range are outliers. The method is not only robust, but simple and non-parametric. An adaptation of the quartile method was proposed **[11]** for trend data where the trends are first transformed to dampen a size masking effect **[10]**.

3.2.2. T-Test in means

3.2.2.1. Principle

The principle of T-Test in means is to compare the means of two different samples at a certain level of confidence. The estimator calculation is based on the standard deviation of each sample. The idea is to sort the measurements of a sample, and then to apply two one-tailed T-Tests for each upper and lower tail of the sample: a test between the whole sample and the sample minus the lower value and a test between the whole sample and the sample minus the upper value.

3.2.2.2. Theoretical approach

Consider two samples - a sample, A, of n_a values, with a mean $\bar{x}_a$ and a standard deviation σ_a and a sample, B, of n_b values, with a mean $\bar{x}_b$ and a standard deviation σ_b.

Define the standard deviation, S_{est}, as:

$$S_{est}^2 = \left(\frac{n_a \times \sigma_a^2 + n_b \times \sigma_b^2}{n_a + n_b - 2} \right)$$

The estimator for the T-Test in mean is then:

$$T = \left(\frac{\left| \bar{x}_a - \bar{x}_b \right|}{S_{est}^2 / n_a + S_{est}^2 / n_b} \right)$$

It is to compare with the theoretical value, $T_{\alpha,\upsilon}$, from the Student law with $\upsilon = n_a + n_b - 2$ degrees of freedom and at a level of confidence $\alpha = 5\%$. If the result of the test is $T > T_{\alpha,\upsilon}$, then the difference in means between the two samples is significant at the level of confidence $1-\alpha = 95\%$.

In our case, sample A is the original sample from OSMOSE measurements and sample B is a sample that contains the same values as A, but includes the suspected outlier. Thus, if $T > T_{\alpha,\upsilon}$, then the suspected value is a significant outlier at the level of confidence $1-\alpha = 95\%$.

3.2.2.3 Results and conclusions

In a similar way to the previous χ^2-Test (Section 3.1.4), the data from the 50 measurements of OSMOSE samples is processed on a separate worksheet. But instead of calculating the estimators as the theoretical approach suggests, we used an Excel function, TINV( ), which can perform a one-tailed T-Test and returns the level of confidence for the two selected samples to have the same mean. Results are shown in Appendix 2**.**

3.2.3. Dixon's Q-Test for discrepancies

3.2.3.1. Principle

In a classic 1950 article **[12]**, Dixon investigated the performance of several statistical tests in terms of their ability to reject bad values in data sets taken from Gaussian populations. The tests investigated included both those which require independent knowledge of the mean or the standard deviation and those which do not require such information. Of the tests included in the latter group, Dixon concluded that tests based on ratios of the range and various subranges were to be preferred as a result of their excellent performance and ease of calculation. The range tests, all of which are closely related, include the following (where the values are ordered such that $x_1 < x_2 < ... < x_{n-1} < x_n$):

1) For a single outlier x_1:

$$r_{10} = \frac{x_2 - x_1}{x_n - x_1} \qquad \left(or \quad \frac{x_n - x_{n-1}}{x_n - x_1} \right)$$

2) For outlier x_1 avoiding x_n:

$$r_{11} = \frac{x_2 - x_1}{x_{n-1} - x_1} \qquad \left(or \quad \frac{x_n - x_{n-1}}{x_n - x_2} \right)$$

3) For outlier x_1 avoiding x_n, x_{n-1}:

$$r_{12} = \frac{x_2 - x_1}{x_{n-2} - x_1} \qquad \left(or \quad \frac{x_n - x_{n-1}}{x_n - x_3} \right)$$

4) For outlier x_l avoiding x_2:

$$r_{20} = \frac{x_3 - x_1}{x_n - x_1} \qquad \left(or \quad \frac{x_n - x_{n-2}}{x_n - x_1} \right)$$

5) For outlier x_1 avoiding x_2 and x_n:

$$r_{21} = \frac{x_3 - x_1}{x_{n-1} - x_1} \qquad \left(or \quad \frac{x_n - x_{n-2}}{x_n - x_2} \right)$$

6) For outlier x_1 avoiding x_2 and x_n, x_{n-1}:

$$r_{22} = \frac{x_3 - x_1}{x_{n-2} - x_1} \qquad \left(or \quad \frac{x_n - x_{n-2}}{x_n - x_3} \right)$$

The parenthetical equations are designed for testing x_n, the highest value rather than the lowest value, x_l.

In Dixon's notation, the first digit in the subscript of each ratio, r_{ij} , refers to the number of possible suspected outliers on the same end of the data as the value being tested, while the second digit indicates the number of possible outliers on the opposite end of the data from the suspected value. Thus, the ratio r_{l0} simply compares the difference between a single suspected outlier (x_l or x_n) and its nearest-neighboring value to the overall range of values in the sample. In other words, it determines the fraction of the total range that is attributable to one suspected outlier. The other ratios are similarly formulated except that they use sub-ranges that are specifically designed to avoid the influence of additional outliers either on the opposite end of the data (r_{ll} and r_{12}), on the same end of the data (r_{20}), or both (r_{21} and r_{22}). Clearly, the latter ratios require larger sample sizes to perform satisfactorily. Dixon subsequently generated critical values for all of these ratios **[13]** for sample sizes and recommended (based on a combination of the relative performance of each ratio and its degree of independence from other outlying values) that, as a general rule, the various ratios be applied as follows **[14]**:

for $3 \le n \le 7$, use r_{l0} ;
for $8 \le n \le 10$, use r_{11} ;
for $11 \le n \le 13$, use r_{21};
for $n \ge 14$, use r_{22}.

The r_{l0} ratio is commonly designated as Q and is generally considered to be the most convenient, legitimate, statistical test available for the rejection of deviant values from a small sample conforming to a Gaussian distribution. It is equally well suited to larger data sets if only one outlier is present. The fact that small data sets are common in analytical testing procedures, in combination with the simplicity of this test, accounts for the fact that the Q test is included in nearly all modern statistical treatises and textbooks designed for use in analytical chemistry **[15]**. Dixon's ratios tables are shown in Appendix 3.

3.2.3.2. Results

As it has been done with χ^2- and T-Tests, the Q-Test was implemented in Excel as part of the macro performing the statistical review. For each sample, the ratios are calculated when possible (there are conditions on the minimum size of the sample for upper ratios), and the ratio corresponding to the sample size is highlighted according to Dixon's suggestion (Section 3.2.3.1). This Q-Test is performed in parallel with the T-Test in order to compare the conclusion of both tests. Results are shown in Appendix 2 as well. It seems that the Q-Test is more efficient than the T-Test, since some measurement results that are somewhat different from all other measurements are systematically rejected, which is the way we would like the sample to be statistically processed.

3.2.4. Conclusions

In characterizing performance, we should characterize errors in a manner that is useful to others who must judge acceptability in their laboratory situations. Comparing the results and interpretation between CEA and ANL is a good example. But the criteria for methods will differ in different laboratories; thus, acceptability will depend on the particular application. Analysis by T-Test is useful, but will not provide specific estimates of errors when proportional error is present. Moreover, this test may be masked by many deviant values at the same tail of the sample. That is why the Q-Test is preferred and widely used to analyze experimental results which are supposed to fit a Gaussian distribution. Dixon's Q-Test is a robust test for outlying values, since it is proven to reject every deviant value, whatever the distribution and as long as the sample has more than 3 measurements. In our study, both tests lead to the same conclusion as in Section 3.1.5.2 - all measurements performed before September 8, 2005 are questionable.

4. Conclusions

Many aspects of the OSMOSE program have been addressed in this report. The experimental part of the project is still on-going at CEA Cadarache and oscillation measurements are forecasted until the year 2009 for OSMOSE and beyond for other programs. Therefore, the automatic data processing method presented in this report was designed to be used again, since it has only been applied to the first series of 50 measurements from FEX 47.

As long as the structure of the output data from MINERVE is the same, the only parameter to modify is the number of measurements, and then the macro runs for itself. All measurements are scrutinized for flaws of the measuring chain, sorted out, calculations and statistical tests for outlying values are run and all results are summarized in a single file.

The statistical review described in this report is not necessarily a new approach, since most of the calculations were already performed at CEA or ANL before. But it was a way to perform a cross-comparison with the previous results. This step is more important as it may seem, since the OSMOSE program involves two laboratories which share results within this framework, but do not necessarily have the same methods.

The ultimate goal of the project is to perform the measurements and provide the highest quality data to the international community as the means to check and improve basic nuclear data. In this end, the cross-comparison of results and the continued study are vital for the improvement and quality assurance of the program.

References

[1] JP. Hudelot, C. Chabert, JP. Chauvin, P. Fougeras, G. Perret, X. Genin, C. Leorier, JM. Moulinier, The OSMOSE Experimental Program in MINERVE for the Qualification of the Integral Cross Sections of Actinides, Journal of Nuclear Science and Technology, Supplement 2, p. 1077-1080, August, 2002.

[2] JP. Hudelot, R. Klann, P. Fougeras, F. Jorion, N. Drin, L. Donnet, OSMOSE : An Experimental Program for the Qualification of Integral Cross Sections of Actinides, Proceedings of the PHYSOR-2004 Topical Meeting, April, 2004.

[3] JP. Hudelot, D. Bernard, Etat de l'art des méthodes de mesure et d'interprétation des experiences d'oscillations dans le réacteur MINERVE : Etude qualitative et quantitative des incertitudes experimentales – Perspectives d'améliorations, Note Technique, CEA / SPEx / LPE, 05-103, January, 2005.

[4] R. Klann, JP. Hudelot, G. Perret, N. Drin, J. Lee, Y. Cao, Final Report of the International Nuclear Energy Research Initiative OSMOSE Project (FY01-FY04), Nuclear Engineering, ANL-04/25, December 17, 2004.

[5] Neuilly, Cetama, Modélisation et estimation des erreurs de mesure, Technique et Documentation, ISBN: 2-7430-0272-7, 1998

[6] P. Albarede, Les mesures de réactivité dans MINERVE, Note Technique, CEA / SPRC/ LEPh, 94-227, 1994.

[7] Mail exchange with JP. Hudelot, from CEA-Cadarache.

[8] I.T. Joliffe, Principal Component Analysis, Springer-Verlag, New York, 1986.

[9] N. Chinnappa, E. Outrata, General survey functiuns design at Statistics Canada, I.S.I. Proceedings, 47th Session, 219-238, 1989.

[10] S. Franklin, M. Brodeur, A practical Application of a Robust Multivariate Outlier Detection Method, Statistics Canada Report, 1997.

[11] M. A. Hidiroglou, JM. Berthelot, Statistical edit and imputation for periodic surveys, Survey Methodology, 12, p. 73-83, 1986.

[12] W. J. Dixon, Ann. Math. Stat., vol. 21, p. 488-506, 1950.

[13] W. J. Dixon, Ann. Math. Stat., vol. 22, p. 68-78, 1951.

[14] W. J. Dixon, Biometrics, vol. 9, p. 74-89, 1953.

[15] D. B. Rorabacher, Statistical Treatment for Rejection of Deviant Values : Critical Values of Dixon's "Q" Parameter and Related Subrange Ratios at the 95% Confidence Level, Analytical Chemistry, vol. 63, No. 2, p. 139-146, January 15, 1991.

Appendix 1 : Statistical review of OSMOSE samples

A1.1 Output file for amplitude (A_i, Aci)

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
1	1	7054	3838.9	1238.1	Unat	9/1/05	2752.5	545.3	35.33	14.07	Refused
7	1	10447	1317.2	1051.9	Unat	9/8/05					
14	1	257	1800.4	1138.1	Unat	9/13/05					
23	1	1330	1580.9	1190.6	Unat	9/27/05					
31	1	5	3730.2	1151.8	Unat	9/30/05					
37	1	113	1505.0	1143.4	Unat	10/4/05					
39	1	17	1249.8	1158.2	Unat	10/5/05					
40	1	44	610.7	1160.7	Unat	10/6/05					
	8	**2408**	**1954**	**1154**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
5	1	61	2434.6	106.3	U234	9/6/05	290.0	420.7	2.76	9.49	Accepted
11	1	85	1460.2	107.7	U234	9/12/05					
19	1	265	1284.5	82.2	U234	9/21/05					
27	1	359	2789.8	117.5	U234	9/29/05					
35	1	390	1389.2	78.8	U234	10/4/05					
	5	**232**	**1872**	**99**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
6	1	24407	3502.5	6386.8	Ure	9/7/05	9025.8	1841.5	24.51	11.07	Refused
16	1	55	7177.5	6535.6	Ure	9/19/05					
20	1	1428	3610.8	6505.2	Ure	9/21/05					
28	1	320	3530.8	6560.9	Ure	9/29/05					
36	1	15375	1739.2	6667.0	Ure	10/4/05					
38	1	3544	1605.0	6602.5	Ure	10/5/05					
	6	**7521**	**3528**	**6543**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
2	1	259	4125.8	359.3	Th232	9/2/05	358.2	537.6	2.67	9.49	Accepted
8	1	7	2173.7	378.0	Th232	9/8/05					
15	1	489	2090.3	353.3	Th232	9/13/05					
24	1	173	1221.0	388.6	Th232	9/27/05					
32	1	506	2395.6	397.9	Th232	9/30/05					
	5	**287**	**2401**	**375**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
3	1	899	3927.8	4531.8	Pu239	9/2/05	913.4	1244.4	2.94	9.49	Accepted
9	1	1002	4670.8	4530.1	Pu239	9/8/05					
17	1	803	9919.5	4590.1	Pu239	9/20/05					
25	1	7	4478.1	4564.3	Pu239	9/28/05					
33	1	944	4460.8	4592.5	Pu239	9/30/05					
	5	**731**	**5491**	**4562**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
4	1	55	723.3	48.8	Pu242	9/6/05	51.5	249.9	0.82	9.49	Accepted
10	1	67	1978.3	64.3	Pu242	9/8/05					
18	1	10	1230.4	53.0	Pu242	9/20/05					
26	1	23	1088.9	51.4	Pu242	9/28/05					
34	1	52	1021.5	63.4	Pu242	9/30/05					
	5	**41**	**1208**	**56**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
42	1	253	729.2	234.5	Np0.1	11/22/05	249.6	324.7	2.31	7.81	Accepted
45	1	126	633.3	229.8	Np0.1	11/23/05					
47	1	191	564.7	204.7	Np0.1	11/23/05					
50	1	178	2519.1	205.2	Np0.1	11/24/05					
	4	**187**	**1112**	**219**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Aci	Sample	Date	S2	Sig_s2	Estim	Chi_inv	Test
41	1	3258	3786.0	3787.4	Np0.6	11/22/05	1541.2	735.5	6.29	7.81	Accepted
44	1	904	634.2	3874.6	Np0.6	11/23/05					
46	1	419	1773.8	3865.0	Np0.6	11/23/05					
49	1	43	949.8	3851.0	Np0.6	11/24/05					
	4	**1156**	**1786**	**3844**	**Mean**						

A1.2 Output file for reactivity worth (DEM1*, Dci)

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test
1	1	20591969	9786042.8	69082.4	Unat	9/1/05	7656761.4	1482138.3	36.16	14.07	Refused
7	1	30078990	3100569.1	59060.1	Unat	9/8/05					
14	1	235098	5035215.5	64059.7	Unat	9/13/05					
23	1	2226596	4430780.4	66036.7	Unat	9/27/05					
31	1	119272	9315257.4	64889.9	Unat	9/30/05					
37	1	327865	3674273.8	63971.9	Unat	10/4/05					
39	1	15907	3685541.2	64670.6	Unat	10/5/05					
40	1	1632	2231840.3	64584.9	Unat	10/6/05					
	8	**6699666**	**5157440**	**64545**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test
5	1	9357	8110491.5	-4436.7	U234	9/6/05	524741.9	1302333.4	1.61	9.49	Accepted
11	1	332692	4631661.2	-4916.7	U234	9/12/05					
19	1	295863	5754742.1	-3796.0	U234	9/21/05					
27	1	620084	7007579.3	-5127.4	U234	9/29/05					
35	1	840973	4954893.9	-3422.9	U234	10/4/05					
	5	**419794**	**6091874**	**-4340**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test

6	1	66271483	8463101.7	354948.2	Ure	9/7/05	18131005.5	3375497.5	26.86	11.07	Refused
16	1	2356416	3137162.1	364623.9	Ure	9/19/05					
20	1	9206	11132322.7	362992.9	Ure	9/21/05					
28	1	28287	3701251.5	363257.1	Ure	9/29/05					
36	1	7279362	2978046.3	365786.9	Ure	10/4/05					
38	1	14710274	1450998.7	366924.3	Ure	10/5/05					
	6	**15109171**	**5143814**	**363089**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test
2	1	704394	10666502.1	20952.3	Th232	9/2/05	1229765.3	1505131.5	3.27	9.49	Accepted
8	1	251	5890158.9	21807.4	Th232	9/8/05					
15	1	1819018	6479799.9	20442.9	Th232	9/13/05					
24	1	905078	2216795.4	22742.9	Th232	9/27/05					
32	1	1490320	7455969.5	23012.4	Th232	9/30/05					
	5	**983812**	**6541845**	**21792**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test
3	1	2087432	9995416.5	252550.7	Pu239	9/2/05	1383052.6	2726713.1	2.03	9.49	Accepted
9	1	409717	11755588.5	253355.4	Pu239	9/8/05					
17	1	38055	16236275.6	253800.4	Pu239	9/20/05					
25	1	2515440	13561583.3	255581.5	Pu239	9/28/05					
33	1	481567	11086752.2	254689.4	Pu239	9/30/05					
	5	**1106442**	**12527123**	**253995**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test
4	1	755279	5050437.6	-925.6	Pu242	9/6/05	285116.1	946740.8	1.20	9.49	Accepted
10	1	241315	7232043.9	-2285.9	Pu242	9/8/05					
18	1	63640	3401954.1	-2046.9	Pu242	9/20/05					
26	1	16235	2114569.7	-1667.3	Pu242	9/28/05					
34	1	63997	4729050.2	-2047.6	Pu242	9/30/05					
	5	**228093**	**4505611**	**-1795**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test
42	1	1419775	3485193.7	14264.3	Np0.1	11/22/05	1200171.0	1193068.3	3.02	7.81	Accepted
45	1	423639	2687629.9	13723.6	Np0.1	11/23/05					
47	1	1197254	1685580.0	11978.6	Np0.1	11/23/05					
50	1	559845	7630175.9	12324.5	Np0.1	11/24/05					
	4	**900128**	**3872145**	**13073**	**Mean**						

Nb	in	Sig_li2	Sig_ei2	Dci	Sample	Date	S2	Sig_s2	Estim	Chi inv	Test

41	1	16014623	9130376.1	-208274.2	Np0.6	11/22/05	7393304.8	2460834.9	9.01	7.81	**Refused**
44	1	1321051	1302568.2	-213425.4	Np0.6	11/23/05					
46	1	4198887	4428348.6	-214325.1	Np0.6	11/23/05					
49	1	645354	2332151.2	-213079.4	Np0.6	11/24/05					
	4	**5544979**	**4298361**	**-212276**	**Mean**						

Appendix 2 : Analysis of OSMOSE samples to detect outlying results.

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
7	1	59060.1	Unat	9/8/05	*0.490*	**0.704**	*0.717*	*0.858*	**false**	0.522	**ok**	left outlier ?
37	1	63971.9	Unat	10/4/05	*0.304*	**0.596**	*0.820*	*0.835*	**false**	0.625	**ok**	right outlier ?
14	1	64059.7	Unat	9/13/05	*0.468*	**0.554**	*0.710*	*0.803*				
40	1	64584.9	Unat	10/6/05								
39	1	64670.6	Unat	10/5/05								
31	1	64889.9	Unat	9/30/05								
23	1	66036.7	Unat	9/27/05								
1	1	69082.4	Unat	9/1/05								
	8	**64545**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
27	1	-5127.4	U234	9/29/05	**0.124**	*0.158*	*0.519*	*1.000*	**ok**	0.684	**ok**	no left outlier
11	1	-4916.7	U234	9/12/05	**0.219**	*0.250*	*0.679*	*1.000*	**ok**	0.617	**ok**	no right outlier
5	1	-4436.7	U234	9/6/05	**0.642**	*0.807*	*0.976*	*1.000*				
19	1	-3796.0	U234	9/21/05								
35	1	-3422.9	U234	10/4/05								
	5	**-4340**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
6	1	354948.2	Ure	9/7/05	**0.672**	*0.742*	*0.767*	*0.859*	**false**	0.419	**false**	**left outlier !**
20	1	362992.9	Ure	9/21/05	**0.095**	*0.289*	*0.585*	*0.627*	**ok**	0.774	**ok**	no right outlier
28	1	363257.1	Ure	9/29/05	**0.560**	*0.689*	*0.872*	*0.983*				
16	1	364623.9	Ure	9/19/05								
36	1	365786.9	Ure	10/4/05								
38	1	366924.3	Ure	10/5/05								
	6	**363089**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
15	1	20442.9	Th232	9/13/05	**0.198**	*0.221*	*0.593*	*1.000*	**ok**	0.637	**ok**	no left outlier
2	1	20952.3	Th232	9/2/05	**0.105**	*0.131*	*0.585*	*1.000*	**ok**	0.680	**ok**	no right outlier
8	1	21807.4	Th232	9/8/05	**0.642**	*0.807*	*0.976*	*1.000*				
24	1	22742.9	Th232	9/27/05								
32	1	23012.4	Th232	9/30/05								
	5	**21792**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
3	1	252550.7	Pu239	9/2/05	**0.266**	*0.376*	*0.584*	*1.000*	**ok**	0.632	**ok**	no left outlier
9	1	253355.4	Pu239	9/8/05	**0.294**	*0.401*	*0.800*	*1.000*	**ok**	0.583	**ok**	no right outlier
17	1	253800.4	Pu239	9/20/05	**0.642**	*0.807*	*0.976*	*1.000*				
33	1	254689.4	Pu239	9/30/05								
25	1	255581.5	Pu239	9/28/05								
	5	**253995**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
10	1	-2285.9	Pu242	9/8/05	**0.175**	*0.385*	*0.386*	*1.000*	**ok**	0.741	**ok**	no left outlier

34	1	-2047.6	Pu242	9/30/05	**0.545**	*0.661*	*0.999*	*1.000*	**false**	0.453	**false**	**right outlier !**
18	1	-2046.9	Pu242	9/20/05	**0.642**	*0.807*	*0.976*	*1.000*				
26	1	-1667.3	Pu242	9/28/05								
4	1	-925.6	Pu242	9/6/05								
	5	**-1795**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
47	1	11978.6	Np0.1	11/23/05	**0.151**	*0.198*	*1.000*	*nb<5*	**ok**	0.667	**ok**	no left outlier
50	1	12324.5	Np0.1	11/24/05	**0.237**	*0.279*	*1.000*	*nb<5*	**ok**	0.626	**ok**	no right outlier
45	1	13723.6	Np0.1	11/23/05	**0.765**	*0.955*	*1.000*	*nb<5*				
42	1	14264.3	Np0.1	11/22/05								
	4	**13073**	**Mean**									

Nb	in	Dci	Sample	Date	r10	r11	r21	r22	Q-Test	t	T-Test	Conclusion
46	1	-214325.1	Np0.6	11/23/05	**0.149**	*0.722*	*1.000*	*nb<5*	**ok**	0.765	**ok**	no left outlier
44	1	-213425.4	Np0.6	11/23/05	**0.794**	*0.933*	*1.000*	*nb<5*	**false**	0.406	**false**	**right outlier !**
49	1	-213079.4	Np0.6	11/24/05	**0.765**	*0.955*	*1.000*	*nb<5*				
41	1	-208274.2	Np0.6	11/22/05								
	4	**-212276**	**Mean**									

Appendix 3: Statistical Tables

A3.1 χ^2-Table:

The Chi-Square (χ^2) Distribution

Degrees of freedom	Area to the Right of the Critical Value									
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169

Donald B. Owen, *Handbook of Statistical Tables*, U.S. Department of Energy (Reading, Mass.: Addison-Wesley, 1962). Reprinted with permission of the publisher.

A3.2 Dixon's tables [15]:

A3.2.1 Q (r_{10}) parameter table:

	confidence level					
N^b	80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
3	0.886	0.941	**0.970**	0.976	0.988	0.994
4	0.679	0.765	**0.829**	0.846	0.889	0.926
5	0.557	0.642	**0.710**	0.729	0.780	0.821
6	0.482	0.560	**0.625**	0.644	0.698	0.740
7	0.434	0.507	**0.568**	0.586	0.637	0.680
8	0.399	0.468	**0.526**	0.543	0.590	0.634
9	0.370	0.437	**0.493**	0.510	0.555	0.598
10	0.349	0.412	**0.466**	0.483	0.527	0.568
11	0.332	0.392	**0.444**	0.460	0.502	0.542
12	0.318	0.376	**0.426**	0.441	0.482	0.522
13	0.305	0.361	**0.410**	0.425	0.465	0.503
14	0.294	0.349	**0.396**	0.411	0.450	0.488
15	0.285	0.338	**0.384**	0.399	0.438	0.475
16	0.277	0.329	**0.374**	0.388	0.426	0.463
17	0.269	0.320	**0.365**	0.379	0.416	0.452
18	0.263	0.313	**0.356**	0.370	0.407	0.442
19	0.258	0.306	**0.349**	0.363	0.398	0.433
20	0.252	0.300	**0.342**	0.356	0.391	0.425
21	0.247	0.295	**0.337**	0.350	0.384	0.418
22	0.242	0.290	**0.331**	0.344	0.378	0.411
23	0.238	0.285	**0.326**	0.338	0.372	0.404
24	0.234	0.281	**0.321**	0.333	0.367	0.399
25	0.230	0.277	**0.317**	0.329	0.362	0.393
29	0.227	0.273	**0.312**	0.324	0.357	0.388
27	0.224	0.269	**0.308**	0.320	0.353	0.384
28	0.220	0.266	**0.305**	0.316	0.349	0.380
29	0.218	0.263	**0.301**	0.312	0.345	0.376
30	0.215	0.260	**0.298**	0.309	0.341	0.372

A3.2.2 r_{11} parameter table:

	confidence level					
N^a	80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
4	0.910	0.955	**0.977**	0.981	0.991	0.995
5	0.728	0.807	**0.863**	0.876	0.916	0.937
6	0.609	0.689	**0.748**	0.763	0.805	0.839
7	0.530	0.610	**0.673**	0.689	0.740	0.782
8	0.479	0.554	**0.615**	0.631	0.683	0.725
9	0.441	0.512	**0.570**	0.587	0.635	0.677
10	0.409	0.477	**0.534**	0.551	0.597	0.639
11	0.385	0.450	**0.505**	0.521	0.566	0.606
12	0.367	0.428	**0.481**	0.498	0.541	0.580
13	0.350	0.410	**0.461**	0.477	0.520	0.558
14	0.336	0.395	**0.445**	0.460	0.502	0.539
15	0.323	0.381	**0.430**	0.445	0.486	0.522
16	0.313	0.369	**0.417**	0.432	0.472	0.508
17	0.303	0.359	**0.406**	0.420	0.460	0.495
18	0.295	0.349	**0.396**	0.410	0.449	0.484
19	0.288	0.341	**0.386**	0.400	0.439	0.473
20	0.282	0.334	**0.379**	0.392	0.430	0.464
21	0.276	0.327	**0.371**	0.384	0.421	0.455
22	0.270	0.320	**0.364**	0.377	0.414	0.446
23	0.265	0.314	**0.357**	0.371	0.407	0.439
24	0.260	0.309	**0.352**	0.365	0.400	0.432
25	0.255	0.304	**0.346**	0.359	0.394	0.426
26	0.250	0.299	**0.341**	0.354	0.389	0.420
27	0.246	0.295	**0.337**	0.349	0.383	0.414
28	0.243	0.291	**0.332**	0.344	0.378	0.409
29	0.239	0.287	**0.328**	0.340	0.374	0.404
30	0.236	0.283	**0.324**	0.336	0.369	0.399

A3.2.3 r_{12} parameter table:

	confidence level					
N[a]	80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
5	0.919	0.960	**0.980**	0.984	0.992	0.996
6	0.745	0.824	**0.878**	0.891	0.925	0.951
7	0.636	0.712	**0.773**	0.791	0.836	0.875
8	0.557	0.632	**0.692**	0.708	0.760	0.797
9	0.504	0.580	**0.639**	0.656	0.702	0.739
10	**0.464**[b]	0.537	**0.594**	0.610	0.655	0.694
11	0.431	0.502	**0.559**	0.575	0.619	0.658
12	0.406	0.473	**0.529**	0.546	0.590	0.629
13	0.387	0.451	**0.505**	0.521	**0.564**[c]	**0.602**[c]
14	0.369	0.432	**0.485**	0.501	0.542	0.580
15	0.354	0.416	**0.467**	0.482	0.523	0.560
16	0.341	0.401	**0.452**	0.467	0.508	0.544
17	0.330	0.388	**0.438**	0.453	0.493	0.529
18	0.320	0.377	**0.426**	0.440	0.480	0.516
19	0.311	0.367	**0.415**	0.429	0.469	0.504
20	0.303	0.358	**0.405**	0.419	0.458	0.493
21	0.296	0.349	**0.396**	0.410	0.449	0.483
22	0.290	0.342	**0.388**	0.402	0.440	0.474
23	0.284	0.336	**0.381**	0.394	0.432	0.465
24	0.278	0.330	**0.374**	0.387	0.423	0.457
25	0.273	0.324	**0.368**	0.381	0.417	0.450
26	0.268	0.319	**0.362**	0.375	0.411	0.443
27	0.263	0.314	**0.357**	0.370	0.405	0.437
28	0.259	0.309	**0.352**	0.365	0.399	0.431
29	0.255	0.305	**0.347**	0.360	0.394	0.426
30	0.251	0.301	**0.343**	0.355	0.389	0.420

A3.2.4 r_{20} parameter table (corrected):

	confidence level					
N[a]	80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
4	0.935	0.967	**0.983**	0.987	0.992	0.996
5	0.782	0.845	**0.890**	0.901	0.929	0.950
6	0.670	0.736	**0.786**	0.800	0.836	0.865
7	0.596	0.661	**0.716**	0.732	0.778	0.814
8	0.545	0.607	**0.657**	0.670	0.710	0.746
9	0.505	0.565	**0.614**	0.627	0.667	0.700
10	0.474	0.531	**0.579**	0.592	0.632	0.664
11	0.449	0.504	**0.551**	0.564	0.603	0.627
12	0.429	0.481	**0.527**	0.540	0.579	0.612
13	0.411	0.461	**0.506**	0.520	0.557	0.590
14	0.395	0.445	**0.489**	0.502	0.538	0.571
15	0.382	0.430	**0.473**	0.486	0.522	0.554
16	0.370	0.418	**0.460**	0.472	0.508	0.539
17	0.359	0.406	**0.447**	0.460	0.495	0.526
18	0.350	0.397	**0.437**	0.449	0.484	0.514
19	0.341	**0.387**[b] (0.379)	**0.427**[b]	0.439	0.473	0.503
20	0.333	**0.378**[b] (0.372)	**0.418**[b]	0.430	0.464	0.494
21	0.326	**0.371**[b] (0.365)	**0.410**[b]	0.422	0.455	0.485
22	0.320	**0.364**[b] (0.358)	**0.402**[b]	0.414	0.447	0.477
23	0.314	**0.358**[b] (0.352)	**0.395**[b]	0.407	0.440	0.469
24	0.309	**0.352**[b] (0.347)	**0.390**[b]	0.401	0.434	0.462
25	0.304	**0.346**[b] (0.343)	**0.383**[b]	0.395	0.428	0.456
26	0.300	**0.342**[b] (0.338)	**0.379**[b]	0.390	0.422	0.450
27	0.296	**0.338**[b] (0.334)	**0.374**[b]	0.385	0.417	0.444
28	0.292	**0.333**[b] (0.330)	**0.370**[b]	0.381	0.412	0.439
29	0.288	**0.329**[b] (0.326)	**0.365**[b]	0.376	0.407	0.434
30	0.285	**0.326**[b] (0.322)	**0.361**[b]	0.372	0.402	0.428

[a] Sample size. [b] Starting with $n = 19$, the r_{20} critical values for both the 90% and 95% confidence levels were calculated from the cubic regression curves fitted to the critical values published by Dixon (*13*) corresponding to the two-tailed 60%, 80%, 96%, 98%, and 99% confidence levels (but omitting the published 90% confidence values). For the 90% confidence level, the values originally published by Dixon are indicated in parentheses underneath the newly generated values. From a comparison of the two sets of values, it is obvious that the critical values in the original table were shifted up one row in the column corresponding to the two-tailed 90% confidence level (see text).

A3.2.5 r_{21} parameter table:

N^a	confidence level 80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
5	0.952	0.976	**0.987**	0.990	0.995	0.998
6	0.821	0.872	**0.913**	0.924	0.951	0.970
7	0.725	0.780	**0.828**	0.842	0.885	0.919
8	0.650	0.710	**0.763**	0.780	0.829	0.868
9	0.594	0.657	**0.710**	0.725	0.776	0.816
10	0.551	0.612	**0.664**	0.678	0.726	0.760
11	0.517	0.576	**0.625**	0.638	0.679	0.713
12	0.490	0.546	**0.592**	0.605	0.642	0.675
13	0.467	0.521	**0.565**	0.578	0.615	0.649
14	0.448	0.501	**0.544**	0.556	0.593	0.627
15	0.431	0.483	**0.525**	0.537	0.574	0.607
16	0.416	0.467	**0.509**	0.521	0.557	0.580
17	0.403	0.453	**0.495**	0.507	0.542	0.573
18	0.391	0.440	**0.482**	0.494	0.529	0.559
19	0.380	0.428	**0.469**	0.482	0.517	0.547
20	0.371	0.419	**0.460**	0.472	0.506	0.536
21	0.363	0.410	**0.450**	0.462	0.496	0.526
22	0.356	0.402	**0.441**	0.453	0.487	0.517
23	0.349	0.395	**0.434**	0.445	0.479	0.509
24	0.343	0.388	**0.427**	0.438	0.471	0.501
25	0.337	0.382	**0.420**	0.431	0.464	0.493
26	0.331	0.376	**0.414**	0.424	0.457	0.486
27	0.325	0.370	**0.407**	0.418	0.450	0.479
28	0.320	0.365	**0.402**	0.412	0.444	0.472
29	0.316	0.360	**0.396**	0.406	0.438	0.466
30	0.312	0.355	**0.391**	0.401	0.433	0.460

A3.2.6 r_{22} parameter table:

N^a	confidence level 80% (α = 0.20)	90% (α = 0.10)	95% (α = 0.05)	96% (α = 0.04)	98% (α = 0.02)	99% (α = 0.01)
6	0.965	0.983	**0.990**	0.992	0.995	0.998
7	0.850	0.881	**0.909**	0.919	0.945	0.970
8	0.745	0.803	**0.846**	0.857	0.890	0.922
9	0.676	0.737	**0.787**	0.800	0.840	0.873
10	0.620	0.682	**0.734**	0.749	0.791	0.826
11	0.578	0.637	**0.688**	0.703	0.745	0.781
12	0.543	0.600	**0.648**	0.661	0.704	0.740
13	0.515	0.570	**0.616**	0.628	0.670	0.705
14	0.492	0.546	**0.590**	0.602	0.641	0.674
15	0.472	0.525	**0.568**	0.579	0.616	0.647
16	0.454	0.507	**0.548**	0.559	0.595	0.624
17	0.438	0.490	**0.531**	0.542	0.577	0.605
18	0.424	0.475	**0.516**	0.527	0.561	0.589
19	0.412	0.462	**0.503**	0.514	0.547	0.575
20	0.401	0.450	**0.491**	0.502	0.535	0.562
21	0.391	0.440	**0.480**	0.491	0.524	0.551
22	0.382	0.430	**0.470**	0.481	0.514	0.541
23	0.374	0.421	**0.461**	0.472	0.505	0.532
24	0.367	0.413	**0.452**	**0.464**[b]	0.497	0.524
25	0.360	0.406	**0.445**	0.457	0.489	0.516
26	0.354	0.399	**0.438**	0.450	**0.482**[b]	0.508
27	0.348	0.393	**0.432**	0.443	0.475	0.501
28	0.342	0.387	**0.426**	0.437	0.469	0.495
29	0.337	0.381	**0.419**	0.431	0.463	0.489
30	0.332	0.376	**0.414**	0.425	0.457	0.483

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