ABSTRACT

This paper examines the calculation and treatment of uncertainty in risk-based allowable outage times (AOTs) for operational control at nuclear power plants, where an AOT is defined as the time that a component or system is permitted to be out of service. The U.S. Nuclear Regulatory Commission (NRC) has explored the possibility of using nuclear power plant's probabilistic risk assessment results to determine component or system AOTs. The analysis and results from previous work prepared for the NRC on determining risk-based AOTs are presented. As part of the discussion, the paper examines the inherent uncertainty in calculating risk-based AOTs and presents the difficulties in calculating these risk-based AOTs. It is noted that care should be taken when dealing with uncertainty analysis results where a time-interval is the outcome of the analysis. In addition, potential improvements in the mechanism of calculating risk-based AOTs are suggested.

KEYWORDS

AOT, risk assessment, uncertainty, Monte Carlo, Boolean difference, correlation
1. INTRODUCTION

At a nuclear power plant, an allowable outage time (AOT) is the length of time that a particular component or system is permitted to be out of service while the plant is operating. This component outage could be induced by many different causes such as random failure, surveillance testing, or preventive maintenance. Since AOTs are controlled by the plant's technical specifications, each particular plant has specific AOTs. In the past, the development of AOTs have not focused on probabilistic risk or reliability considerations. Consequently, it has been suggested that modern risk and reliability techniques be used to evaluate component AOTs at nuclear power plants. Since these risk-based AOTs have the potential for improving operations at nuclear power plants, the Nuclear Regulatory Commission (NRC) has evaluated the possible use of risk-based AOTs for operational control. One of the reports prepared for the NRC, *Feasibility Assessment of a Risk-Based Approach to Technical Specifications*, recommended initiating a pilot program to evaluate real-time, risk-based AOTs. Reference 4 provides the basis for analyzing risk-based AOTs in this paper.

The analysis and results of the previous work on calculating AOTs are summarized in Section 2. Section 3 presents the limitations of the previous work and investigates, in detail, the complications that could result from not addressing the inherent uncertainty in calculating risk-based AOTs. Section 4 presents potential methods for calculating risk-based AOTs where uncertainty is incorporated. Lastly, Section 5 presents results and one recommended method for calculating risk-based AOTs.

2. OVERVIEW OF PREVIOUS WORK

In reference 4, Atefi and Gallagher suggested that a risk-based AOT be calculated using plant-specific probabilistic risk assessment (PRA) results and the equation:

\[ \Delta CD \cdot T \leq B \] ,

(1)
where $\Delta CD$ is the increase in the plant core damage frequency (CDF) caused by the component being out of service, $T$ is the AOT, and $B$ is a dimensionless safety limit. The $\Delta CD$ term would be determined by using a plant's PRA results from a risk analysis code such as SAPHIRE. Since taking components out of service will increase the mean plant risk, and since this positive increase in core damage frequency is of concern, the $\Delta CD$ term is defined by:

$$\Delta CD = CD_{after \ modification} - CD_{before \ modification}.$$  \hspace{1cm} (2)

where $CD_{after \ modification}$ is the core damage frequency after the AOT component(s) are removed from service and $CD_{before \ modification}$ is the nominal core damage frequency.

The definition of the term $B$ from Equation (1) is stated by Atefi and Gallagher as "...the highest acceptable CDF integrated over the duration $T$, which is specified by the AOT." Using a limit of $1 \times 10^{-4}$/year for the CDF, a value of $5.0 \times 10^{-7}$ for the constant $B$ was determined by Atefi and Gallagher. This value for $B$ does not have any regulatory significance and is used only to demonstrate the potential of risk-based AOTs. What is important though is to recognize that constant $B$ is assumed to be a fixed value that represents the absolute upper-limit of the safety-envelope for the risk-based AOT calculation. It should be noted that earlier AOT work generally just attempted to estimate the risk from the AOT rather than specifically contrast the risk level with a dimensionless safety limit as in Equation 1. Also, the focus of this paper is in the treatment of the $\Delta CD$ term as defined in Equation (2). The specific calculation of the individual terms in Equation (2), i.e., the core damage frequency either before or during the AOT, is not address. Details of this calculation can be found in the risk and reliability literature. Further, issues such as the quality of the risk assessment, the overall cumulative risk from multiple outages during an operational cycle, and risk trade-offs (e.g., shutting down versus continuing operation) are not explored. These topics can be found in the risk and reliability literature by authors such as Vesely and Mankamo, Kim, and Samanta.
Using Equation (1), several AOTs were calculated (as a demonstration) for the Surry nuclear power plant and were presented in a workshop on living probabilistic safety analyses by M. Wohl. These calculated AOTs are shown in Table 1. Included in Table 1 are the CDF values and a comparison of the risk-based AOTs with the plant's technical specifications. These results will be used as the basis to show the limitations in the risk-based AOT calculations if uncertainties are not correctly accounted for in the analysis.

**Table 1.** Comparison of the calculated risk-based AOTs and the technical specification AOTs for the Surry nuclear power plant (Reference 7).

<table>
<thead>
<tr>
<th>Case</th>
<th>Component/System unavailable</th>
<th>Core damage frequency after modification (per year)</th>
<th>ΔCD (per year)</th>
<th>Risk-based AOT (hours)</th>
<th>Technical specification AOT (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None (Base Case)</td>
<td>$2.5 \times 10^{-5}$</td>
<td>0</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>One Accumulator</td>
<td>$5.2 \times 10^{-4}$</td>
<td>$5.0 \times 10^{-4}$</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>One LPI Pump</td>
<td>$3.2 \times 10^{-5}$</td>
<td>$7.5 \times 10^{-6}$</td>
<td>584</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>Motor Driven AFW Pump 3A</td>
<td>$3.6 \times 10^{-5}$</td>
<td>$1.1 \times 10^{-5}$</td>
<td>371</td>
<td>72</td>
</tr>
<tr>
<td>5</td>
<td>Motor Driven AFW Pump 3B</td>
<td>$4.4 \times 10^{-5}$</td>
<td>$1.9 \times 10^{-5}$</td>
<td>223</td>
<td>72</td>
</tr>
<tr>
<td>6</td>
<td>Turbine Driven AFW Pump</td>
<td>$7.2 \times 10^{-5}$</td>
<td>$4.7 \times 10^{-6}$</td>
<td>92</td>
<td>72</td>
</tr>
<tr>
<td>7</td>
<td>Two Motor Driven AFW Pumps</td>
<td>$5.6 \times 10^{-5}$</td>
<td>$3.1 \times 10^{-5}$</td>
<td>141</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>One Motor Driven (pump 3B) and the Turbine Driven AFW Pumps</td>
<td>$6.8 \times 10^{-4}$</td>
<td>$6.6 \times 10^{-4}$</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>Diesel Generator 01</td>
<td>$2.7 \times 10^{-4}$</td>
<td>$2.5 \times 10^{-4}$</td>
<td>16</td>
<td>168</td>
</tr>
<tr>
<td>10</td>
<td>Diesel Generator 03</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$2.8 \times 10^{-4}$</td>
<td>16</td>
<td>168</td>
</tr>
<tr>
<td>11</td>
<td>Diesel Generator 01 and 03</td>
<td>$8.4 \times 10^{-3}$</td>
<td>$8.4 \times 10^{-3}$</td>
<td>0.5</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>Diesel Generator 03 and AFW 3A</td>
<td>$5.1 \times 10^{-4}$</td>
<td>$4.9 \times 10^{-4}$</td>
<td>7</td>
<td>*</td>
</tr>
<tr>
<td>13</td>
<td>Diesel Generator 03 and Turbine Driven AFW</td>
<td>$6.8 \times 10^{-4}$</td>
<td>$6.6 \times 10^{-4}$</td>
<td>7</td>
<td>*</td>
</tr>
</tbody>
</table>

* The plant technical specifications do not address this combination of component outages.
3 LIMITATIONS OF PREVIOUS AOT WORK

While the calculated risk-based AOTs shown in Table 1 are straightforward, the inherent uncertainty in the AOT values is not mentioned by Atefi and Gallagher or Wohl. Since the CDF has uncertainty, any calculation that incorporates a CDF has an uncertainty associated with it. Consequently, ignoring the uncertainty of the calculated AOT may result in indefensible risk-based safety goals.

While much of the past work in the area of risk-based AOT determination has not dealt with the intricacies of the associated uncertainty, it should be realized up-front that treatment of uncertainty is a crucial facet of the analysis for one reason; the AOT calculation involves the difference of two parameters (e.g., core damage frequency) that are highly correlated. As will be demonstrated later in this paper by way of example, simply subtracting two parameters without regard to the degree of their correlation can lead to nonsensical results. But, it is this flawed process that has been used in the past to demonstrate the calculation of AOTs.

Although Wohl did not indicate the type of statistical parameter (e.g., mean, median) for the values listed in Table 1, it is assumed that the CDF values are medians (i.e., 50th percentile) since the base case value closely matches the median values given in the Surry NUREG/CR-4550 report. But, even though median CDF values were used to calculate the AOTs, the resulting AOTs presented in Table 1 are not median values. When evaluating the definition of the constant B, it is assumed that B would represent the highest limit of the CDF integrated over the duration of the calculated AOT. If we are only about 50% sure of being below the highest limit of CDF (assuming the AOT is calculated using median values), the AOTs, as calculated and presented by Atefi and Gallagher and Wohl, are not useful from a safety and regulatory standpoint. The reason that this interpretation of the safety limit B is not useful is that B is considered to be a target rather than a safety limit. If the regulator were to consider the safety limit B as being a true limit and is highly desirable to keep the operational risk level below the limit, then the AOT calculation presented in Equation 1 could be useful. Unfortunately though, for this second interpretation we cannot treat B as a middle, mean, or median type of value (i.e., a target) but rather an upperbound. Treating the safety limit B
as an upperbound in Equation 1 then forces the calculated AOT duration to become much smaller than if the limit were treated as a target. And, as we will demonstrate later in this paper, the resulting AOT duration may be smaller than the existing, deterministic technical-specification AOT.

Since the AOT calculations given by Atefi and Gallagher are useful to illustrate the potential for risk-based operational control, some method of risk-based control which incorporates the inherent uncertainty should be available. One solution would be to use Equations (1) and (2) and propagate the inherent uncertainty by using a Monte Carlo simulation. This technique was used to duplicate the calculations by Wohl and is discussed below.

Before a Monte Carlo simulation could be performed, the uncertainty in the core damage parameters was needed. Once again, looking at the Surry NUREG/CR-4550 report, it was determined that the base case CDF had an error factor (i.e., the ratio of the 95th percentile to the 50th percentile) of approximately 5.8. Consequently, it was assumed for the analysis in this paper that the base case CDF was lognormally distributed with a median of $2.5 \times 10^{-5}$/year and an error factor of 5.8. This assumption results in a base case lognormal variate with a mean of $4.4 \times 10^{-5}$/year and standard deviation of $6.5 \times 10^{-5}$/year. The CDF obtained after setting a component to "failed" (for the AOT calculation) was also assumed to be lognormally distributed with a median of that given in Table 1 and an error factor of 5.8 (for the simple, illustrative analyses in this paper, it is assumed that the error factors do not change from case to case which, in general, is not a valid assumption). Table 2 lists the uncertainty parameters for the different AOT cases to be evaluated. These parameters were used with Equations (1) and (2) in the Monte Carlo simulation package @RISK. During the simulation, each case was sampled 10,000 times.

Each of the resulting AOTs from the Monte Carlo analysis has a specific distribution. As an example, the resulting AOT probability density for Case 7 is shown in Figure 1. The @RISK package is able to calculate the statistical parameters such as the mean and median of the distribution for each case. Three of these distribution parameters (5th percentile, 50th percentile, and the mean) for each case are presented in Table 3.
### Table 2. Uncertainty parameters for calculating AOTs for the Surry nuclear power plant.

<table>
<thead>
<tr>
<th>Case</th>
<th>Component/System unavailable</th>
<th>Median core damage frequency after modification (per year)</th>
<th>Mean core damage frequency after modification (per year)</th>
<th>Standard deviation of core damage (per year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None (Base Case)</td>
<td>$2.5 \times 10^{-5}$</td>
<td>$4.4 \times 10^{-5}$</td>
<td>$6.5 \times 10^{-5}$</td>
</tr>
<tr>
<td>2</td>
<td>One Accumulator</td>
<td>$5.2 \times 10^{-4}$</td>
<td>$9.2 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>3</td>
<td>One LPI Pump</td>
<td>$3.2 \times 10^{-5}$</td>
<td>$5.7 \times 10^{-5}$</td>
<td>$8.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>4</td>
<td>Motor Driven AFW Pump 3A</td>
<td>$3.6 \times 10^{-5}$</td>
<td>$6.4 \times 10^{-5}$</td>
<td>$9.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>5</td>
<td>Motor Driven AFW Pump 3B</td>
<td>$4.4 \times 10^{-5}$</td>
<td>$7.8 \times 10^{-5}$</td>
<td>$1.1 \times 10^{-4}$</td>
</tr>
<tr>
<td>6</td>
<td>Turbine Driven AFW Pump</td>
<td>$7.2 \times 10^{-5}$</td>
<td>$1.3 \times 10^{-4}$</td>
<td>$1.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>7</td>
<td>Two Motor Driven AFW Pumps</td>
<td>$5.6 \times 10^{-5}$</td>
<td>$9.9 \times 10^{-5}$</td>
<td>$1.4 \times 10^{-4}$</td>
</tr>
<tr>
<td>8</td>
<td>One Motor Driven (pump 3B) and the Turbine Driven AFW Pumps</td>
<td>$6.8 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-3}$</td>
<td>$1.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>9</td>
<td>Diesel Generator 01</td>
<td>$2.7 \times 10^{-4}$</td>
<td>$4.8 \times 10^{-4}$</td>
<td>$7.0 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>Diesel Generator 03</td>
<td>$3.0 \times 10^{-4}$</td>
<td>$5.3 \times 10^{-4}$</td>
<td>$7.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>11</td>
<td>Diesel Generator 01 and 03</td>
<td>$8.4 \times 10^{-3}$</td>
<td>$1.5 \times 10^{-2}$</td>
<td>$2.2 \times 10^{-2}$</td>
</tr>
<tr>
<td>12</td>
<td>Diesel Generator 03 and AFW 3A</td>
<td>$5.1 \times 10^{-4}$</td>
<td>$9.0 \times 10^{-4}$</td>
<td>$1.3 \times 10^{-3}$</td>
</tr>
<tr>
<td>13</td>
<td>Diesel Generator 03 and Turbine</td>
<td>$6.8 \times 10^{-4}$</td>
<td>$1.2 \times 10^{-3}$</td>
<td>$1.8 \times 10^{-3}$</td>
</tr>
</tbody>
</table>
Figure 1. AOT distribution from Monte Carlo analysis for Case 7.
<table>
<thead>
<tr>
<th>Case</th>
<th>Component/System unavailable</th>
<th>5th percentile AOT (hours)</th>
<th>50th percentile AOT (hours)</th>
<th>Mean AOT (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None (Base Case)</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>One Accumulator</td>
<td>1.1</td>
<td>8.4</td>
<td>110</td>
</tr>
<tr>
<td>3</td>
<td>One LPI Pump</td>
<td>-1,000</td>
<td>35</td>
<td>720</td>
</tr>
<tr>
<td>4</td>
<td>Motor Driven AFW Pump 3A</td>
<td>-960</td>
<td>39</td>
<td>180</td>
</tr>
<tr>
<td>5</td>
<td>Motor Driven AFW Pump 3B</td>
<td>-790</td>
<td>46</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>Turbine Driven AFW Pump</td>
<td>-460</td>
<td>40</td>
<td>-160</td>
</tr>
<tr>
<td>7</td>
<td>Two Motor Driven AFW Pumps</td>
<td>-660</td>
<td>44</td>
<td>2,700</td>
</tr>
<tr>
<td>8</td>
<td>One Motor Driven (pump 3B) and the Turbine Driven AFW Pumps</td>
<td>0.90</td>
<td>6.7</td>
<td>-220</td>
</tr>
<tr>
<td>9</td>
<td>Diesel Generator 01</td>
<td>-21</td>
<td>16</td>
<td>-1.5</td>
</tr>
<tr>
<td>10</td>
<td>Diesel Generator 03</td>
<td>-13</td>
<td>15</td>
<td>58</td>
</tr>
<tr>
<td>11</td>
<td>Diesel Generator 01 and 03</td>
<td>0.09</td>
<td>0.52</td>
<td>0.99</td>
</tr>
<tr>
<td>12</td>
<td>Diesel Generator 03 and AFW 3A</td>
<td>1.1</td>
<td>8.6</td>
<td>3.0</td>
</tr>
<tr>
<td>13</td>
<td>Diesel Generator 03 and Turbine</td>
<td>0.96</td>
<td>6.7</td>
<td>23</td>
</tr>
</tbody>
</table>
Comparing the risk-based AOT results presented in Table 3 to those results shown in Table 1, it is apparent that incorporating the uncertainty of the CDF through simple Monte Carlo simulation dramatically changes the outcome of the AOT calculation. For example, some of the mean AOT values turn out to be negative, which, in the context of this analysis, does not make sense. The reasoning behind obtaining a negative AOT is that the \( \Delta CD \) term [i.e., Equation (2)] may be negative since the Monte Carlo analysis is subtracting two random variates. A random sample for the \( CD_{\text{before modification}} \) term could be larger than a random sample for the \( CD_{\text{after modification}} \) term for any particular iteration, thereby resulting in a negative value for \( \Delta CD \). While from a practical standpoint a negative CD value is nonsensical, incorporating the uncertainty of the CDF terms in a normal fashion (i.e., Monte Carlo simulation) could result in useless results. This element of uncertainty in the before and after terms for CDF signifies the fact that uncertainty must be evaluated in order to have defendable, meaningful, risk-based AOT results.

But, even for those AOT distributions which are positive, the decision must be made as to which parameter of the three (5th percentile, 50th percentile, or mean) should be used to control AOTs for safety-related components. Using the median or mean values does not imply an upper limit to the safety envelope (i.e., back to the target concept). The 5th percentile, on the other hand, could be thought of as a defendable bound on the safety limit for a particular AOT. The 5th percentile could be used for the AOT since it is desirable to keep operational risk below a predefined safety limit. By using the 5th percentile of the resulting AOT distribution, we can claim that there is 95% probability that the resulting risk from the AOT is below the safety limit defined by the constant B. Unfortunately though, the 5th percentile AOT value may be much smaller than either the mean or median values, which may result in very short AOTs. For example, from Table 3, for Case 2, the 5th percentile value of the AOT is 1.1 hours while the median is 8.4 hours. Using the 5th percentile would imply that the new risk-based AOT for having one accumulator out of service would be 1.1 hours instead of the 4 hours given by the technical specifications or 8 hours given by Wohl.
4 OTHER METHODS OF EVALUATING AOT UNCERTAINTY

It has been shown that previous risk-based AOT calculations are suspect in their usefulness since the analysis lacked an uncertainty evaluation. While the idea of having a risk-based AOT may be a reasonable concept, actual risk-based AOT usage may be limited because of the inherent uncertainty in the $\Delta CD$ term. Simple (uncorrelated) Monte Carlo analysis was shown to be prone to such nonsensical results as negative AOT values.

Fortunately, other methods exist which may better evaluate the risk-based AOT while still incorporating the inherent uncertainty. Potential methods which may improve the uncertainty analysis woes for risk-based AOTs include:

1. Run the AOT Monte Carlo simulation using correlated $CD_{\text{before modification}}$ and $CD_{\text{after modification}}$ distributions. Using correlated distributions will utilize the same percentile value from the two CD distributions, thereby resulting in predominantly positive $\Delta CD$ values.

2. Use a Taylor series expansion to evaluate the AOT expression and then fit the resulting statistical parameters to an appropriate (possibly lognormal) distribution. Using a distribution such as a gamma or lognormal for the AOT ensures that the lower bound will be a positive value.

3. Use the 95th percentile values (instead of the median values) for the $\Delta CD$ calculation in Equation (2) to obtain an approximate 95% probability level for the calculated AOT.

4. Evaluate the logic cut sets which comprise the $\Delta CD$ term in order to incorporate the uncertainty on the Boolean difference (i.e., the Boolean subtraction on the term $CD_{\text{after modification}} - CD_{\text{before modification}}$).

The first potential method is justified because the $CD_{\text{before modification}}$ and $CD_{\text{after modification}}$ terms in the $\Delta CD$ calculation probably should be correlated. Using a statistical correlation coefficient of 1.0 for the Monte Carlo simulation will result in the calculation of primarily positive AOT values.
The second potential method has two different problems. First, the Taylor series expansion may not treat the non-linear AOT equation adequately without using higher-order statistical parameters (e.g., skewness and kurtosis). For linear equations, such as the calculations of ΔCDF, the Taylor series expansion may be adequate enough. Second, no statistical justification may be available to explain the use of fitting the AOT parameters to a gamma or lognormal distribution.

The third potential method is limited in that an approximate probability level (i.e., approximately 95%) is obtained from the AOT calculation. The calculated 95th percentile for the calculated AOT is not equal to the approximate value obtained by evaluating the AOT equation using the 95th percentile values for the CDF parameters. Thus, the potential error between the approximate and calculated probability values should be evaluated carefully before this method is used.

The fourth method should provide the best method for incorporating the uncertainty in the AOT calculation since it provides the exact representation of the ΔCD term. This method is discussed in detail in the next section. Unfortunately, it is beyond the scope of this paper to explore all four analysis methods in detail. Although the fourth method is expected to provide the best model for estimating risk-based AOTs, the first three methods should not be ignored. One of the first three methods may provide reasonable AOT results for a particular situation. Specific use of the first three methods is left to the judgement of the reader or for further study.
5 EXACT METHOD FOR AOT CALCULATION

The exact method evaluates the logic cut sets that comprise the ΔCD term in order to incorporate the uncertainty on the Boolean difference. Figure 2 provides a Venn diagram showing what the ΔCD term is with respect to the before and after CD terms. This figure can be generalized in order to determine the equation representing the Boolean difference. Assuming that A represents the CD_{after modification} probability space on the Venn diagram while B represents the CD_{before modification} probability space, the equation for the Boolean difference A - B can be shown to be

\[ A - B = A \cap \overline{B} \]  

(3)

where the \( \overline{B} \) represents the complement of B.

As a simple example of how this Boolean difference works with core-damage cut sets, assume that the cut sets before a component is removed (i.e., logically failed) are

\[ A B C + \]

\[ A D . \]

where the "+" signifies that the two cut sets are logically OR'ed together and "A B C" represents failure of component A AND component B AND component C. Note that this example and the calculations illustrated earlier in this paper ignore potential modifications that may be needed to parameters such as common-cause failures.

If the AOT for component A is to be determined, the Boolean difference needs to be evaluated given that component A is removed from the cut sets. Thus, our modified cut sets will look like

\[ B C + \]

\[ D . \]
Figure 2. Illustration showing the Boolean difference of set A from set B.

Using the before and modified cut sets with Equation (3), we can obtain

\[
\Delta CD = (BC + D) \cap (ABC + AD)
\]
\[
= (BC + D) \cap (ABC \cap AD)
\]
\[
= (BC + D) \cap (\overline{A} + \overline{B} + \overline{C}) \cap (\overline{A} + \overline{D})
\]
\[
= (BC + D) \cap (\overline{A} + AD + BA + BD + CD)
\]
\[
= (BC + D) \cap (\overline{A} + BD + CD)
\]
\[
= BC\overline{A} + D\overline{A},
\]

which can be rewritten as

\[
\Delta CD = \overline{A}(BC + D).
\] (4)
It turns out that

\[ \overline{A}(BC + D) = \overline{A} \Phi , \]

where \( \Phi \) represents the cut sets modified by following two steps:

1. All cut sets not including the "removed component" are pruned from the list.
2. The "removed component" basic event is then itself removed from the remaining cut sets from step 1.

Consequently, the resulting expression for the \( \Delta CD \) term is

\[ \Delta CD = \overline{\text{Removed Component}} \Phi , \]

and, for this equation, only the uncertainty for the complement of the removed component and the modified cut sets will need to be evaluated. Typically, this uncertainty will be smaller than the uncertainty inherent in subtracting the before and after cut sets. It is for this reason why this method may be preferred when calculating risk-based AOTs. One current limitation to this method is that PRA analysis tools do not have this calculation incorporated into the software. This situation could change once developers of PRA analysis codes develop the appropriate calculation algorithms for evaluating changes in a plant core damage frequency. This method is useful not only in calculating risk-based AOTs but other risk-based parameters that use the \( \Delta CD \) term (e.g., regulatory cost/benefit analysis, operational events assessment).

One important limitation in the exact calculation as illustrated in Equation (3) is for the case where multiple components are removed from service at the same time. For this case, the analysis of the Boolean difference on the many cut sets that exist within typical PRA results rapidly yields a difficult numeric problem. The solution to this problem is left for further research. An additional, although minor, issue is that concerning truncation when dealing with a large quantity of cut sets. Since the methodology for the
exact calculation works on the level of cut sets, the traditional issues faced with truncating the cut sets are
still relevant.

6 EXAMPLE OF AOT CALCULATION

To illustrate the uncertainty difference between the exact method and the simple subtraction of a
before CD from the after CD, a Taylor series expansion about the mean (using the rare event approximation
and the CD cut sets) was performed. For this illustration, the example problem discussed previously will be
used. The two $\Delta CD$ equations used are:

$$
\Delta CD = CD_{\text{after modified}} - CD_{\text{before modification}} \quad \text{(simple method)}
$$

$$
\Delta CD_{\text{exact}} = \bar{A} \Phi \quad \text{(exact method)}
$$

where:

- $CD_{\text{before modification}}$ = the cut sets before removing the component
  
  = $ABCD + AD$

- $CD_{\text{after modification}}$ = the cut sets after removing the component
  
  = $BCD$

- $\bar{A}$ = the complement of the event representing the removed component

- $\Phi$ = the cut sets modified via the two-step process discussed previously.
  
  = $BCD$

The mean and standard deviation used for each of the events shown in the equations are listed in Table 4
below.
Before performing the calculation represented by Equation (5), the “global” CD parameters need to be evaluated. To do this evaluation, each CD term was evaluated using a second-order Taylor series expansion about the mean value (using the cut sets and the rare-event approximation). The results of this step were found to be: the mean and standard deviation for CD_{before modification} are 5.5 \times 10^{-3} and 1.6 \times 10^{-3}, respectively, and the mean and standard deviation for CD_{after modification} are 5.5 \times 10^{-4} and 7.5 \times 10^{-3}, respectively. The mean and standard deviation for these two “global” CD terms were then used as inputs in the Taylor series evaluation of Equation (5).

From the Taylor series expansion, the resulting mean and standard deviation for Equations (5) and (6) are shown in Table 5, where Equation 5 is the simple method (ΔCD) and Equation 6 is the exact method (ΔCD_{exact}).

<table>
<thead>
<tr>
<th>Equations</th>
<th>mean</th>
<th>standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 (ΔCD)</td>
<td>5.0 \times 10^{-3}</td>
<td>7.7 \times 10^{-3}</td>
</tr>
<tr>
<td>6 (ΔCD_{exact})</td>
<td>5.0 \times 10^{-3}</td>
<td>6.9 \times 10^{-3}</td>
</tr>
</tbody>
</table>

As shown in Table 5, the uncertainty (as measured by the standard deviation) for the exact method (Equation 6) in calculating the ΔCD is lower (about 10%) than using Equation 5. The lower uncertainty is attributed to the ΔCD being calculated using the exact Boolean equation. Simply subtracting the two CDs allows for a larger uncertainty to be incorporated into the final ΔCD. While the uncertainty difference is small for this
simple problem, more complex models may demonstrate larger differences. The demonstration of this is left for future exploration.

7 CONCLUSION

The paper discusses the importance of incorporating uncertainty into risk-based AOT determinations. By identifying the uncertainty in Equation 1 and propagating this uncertainty through the equation as demonstrated, the result will allow the analyst some means of presenting defensible results. Only after incorporating the uncertainty can it be justified to increase the AOT of a particular component if the results show that, by allowing the component to be out of service for AOT duration, the risk will not be above a predetermined safety limit. While much of the past work in the area of risk-based AOT determination has not dealt with the intricacies of the associated uncertainty, the treatment of uncertainty is a crucial facet of the analysis for one reason; the AOT calculation involves the difference of two parameters (e.g., core damage frequency) that are highly correlated. As was demonstrated, simply subtracting two parameters without regard to the degree of their correlation can lead to nonsensical results.

It was also noted that, given a safety value such as the parameter B from Equation 1, this value should be treated as a limit rather than a target. Using the safety value as a limit rather than a target would ensure that the risk level enumerated by the value is not exceeded (or is unlikely to be exceeded). Previous AOT research work has not made this distinction between the two concepts of a safety limit versus a target.

The paper also looks at various ways that uncertainty can be incorporated and quantified for the AOT equation. Of the different methods, one method utilizes an exact $\Delta CD$ calculation. This exact calculation uses the concept of Boolean difference of cut sets in order to evaluate the uncertainty when two sets of cut sets are subtracted (e.g., CD before and after). The exact method was used to show how the uncertainty from the analysis results can be decreased (i.e., the overall distribution width is decreased) when compared to using a subtraction of two “global” CD terms. And, since the AOT duration value is derived from the resulting lower bound on the AOT uncertainty distribution, a decrease in the uncertainty distribution will
yield an increase in the AOT duration. But, limitations in this method (mainly complexity when dealing with multiple, simultaneous component outages) were also noted and are left for future work.

And lastly, to have defensible AOT results, a proper uncertainty quantification (using an appropriate risk model) of the results must be performed. Without this uncertainty quantification, an analyst or regulatory agent has little basis on which a decision can be made. And, since a risk model underlies the results of the AOT analysis, deficiencies in the risk model itself could cause the resulting AOT calculations to be suspect. Investing the resources to build a risk model and then subsequently use that model as part of plant operation obligates the users to ensure the quality of the model.
8 REFERENCES


